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Original Article

# Setup cost reduction in an integrated production inventory system for defective items with service level constraint and delay in payments

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**ABSTRACT**
In this article, we have developed a two level supply chain model for defective items with controllable lead time in an imperfect production process. Trade credit offered by the supplier to the retailer is considered. The lead time demand follows a normal distribution and the lead time is crashed. The vendor's setup cost is reduced by an extra added cost. A mathematical model is derived to obtain the optimal number of shipments delivered from vendor to buyer in a production cycle, the order quantity, lead time and setup cost with the objective of minimizing the total expected cost. The theory developed in this article is illustrated using a numerical example. Managerial insights and the effect of key parameters are studied through sensitivity analysis to analyze the behavior of model.

**KEYWORDS** Supply chain · imperfect production processes · Permissible delay in payment · Service Level Constraint

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## 1. INTRODUCTION

Inventory control is the activity which organizes the availability of items to the customers. Managing inventory of an organization plays a vital role in the supply chain. A supply chain is a network of facilities that procure raw materials, transform them into intermediate goods and then to final products, and finally deliver the products to customers through a distribution system that includes an inventory system. Thus, it spans procurement, manufacturing, and distribution with an effective inventory management as one key element. To fill orders efficiently, it is necessary to understand the linkages and interrelationships of all the key elements of the supply chain. In addressing this issue, many researchers such as Ghare and Schrader (1963), Covert and Philip (1973), Ouyang et al. (2002), Teng (2002), Udayakumar and Geetha (2014, 2016), Jaggi et al. (2015), Geetha and Udayakumar (2015, 2016) developed the economic order quantity (EOQ) model under various assumptions. Recently, Udayakumar and Geetha (2017) established an EOQ model with a two level storage facility under trade credit policy.

Therefore, integrated management of the supply chain has become a key success factor for some of today's leading companies. Coordination between the two different business entities (vendor and buyer) is an important mean to increase the competitive advantage because coordinative strategy lowers the supply chain cost and increases their revenue. The cooperation between vendor and buyer for improving the performance of inventory control has received a great deal of attention, and the integration approach has been studied for years. The study of the integrated supplier-retailer inventory model was first advocated by Goyal (1996). Subsequently, many researchers investigated this issue under various assumptions. Hill (1997) developed an integrated single-vendor single-buyer production inventory model with a generalized policy. Sarmah et al. (2006) gave an intended literature review to cover the entire gamut of supply chain coordination mechanism. Yao et al. (2007) established the supply chain integration model based on vendor managed inventory. Giri and Sharma (2014) gave manufacturer’s pricing strategies in cooperative and non-cooperative advertising supply chain under retail competition. Rajkumar et al. (2016) presented a doctoral dissertation in logistics and supply chain management.

It is common yet unrealistic to assume that all the units produced are of good quality. The classical Economic Order Quantity (EOQ) model assumes that the items produced are of perfect quality, which is usually not the case in real production. Huang (2001) developed an integrated inventory model for supplier and retailer with defective items. In his article, he incorporates the view of the integrated supplier-retailer approach into the inventory model with imperfect items to determine the optimum ordering quantity and number of deliveries per order. Goyal et al. (2003) gave a simple integrated production policy of an imperfect item for vendor and buyer. Hsu & Hsu (2012 & 2013) derived an integrated single-vendor single-buyer production-inventory model for items with imperfect quality and inspection errors. Hsu & Hsu (2014) gave a supplement to an EOQ model with imperfect quality items, inspection errors, shortage backordering, and sales return. Kundu and Chakraborti (2015) derived an integrated multi stage supply chain inventory model with imperfect production process.

In some practical situations, lead time and ordering cost can be controlled and reduced in various ways. When the demand during the cycle period is not deterministic but is stochastic, lead time becomes an important issue and its control leads to many benefits. The lead time can be reduced by an additional crashing cost. In this direction, Goyal (2003) gave a note on controlling the controllable lead time component in the integrated inventory model. Ouyang et al. (2007) established an integrated vendor-buyer inventory model with quality improvement and lead time reduction. Annadurai and Uthayakumar (2010) gave the model with investment in setup cost with defective items.

Trade credit is an essential tool for financial growth for many businesses. In order to encourage sales, such a credit is given. During this credit period the retailer can accumulate and earn interest on the encouraged sales revenue. In spite of the extension period the supplier charges interest on the unpaid balance. Hence, the permissible delay period indirectly reduces the cost of holding cost. Also trade credit offered by the supplier encourages the retailer to buy more products. Hence, the trade credit plays a major role in inventory control for both the supplier as well as the retailer. The integration between the vendor and the buyer for improving the performance of inventory control with permissible delay in payment plays the major role to minimize the joint total cost for any business firm. Ouyang et al. (2008) established an optimal strategy for an integrated system with variable production rate when the freight rate and the trade credit are both linked to the order quantity. Ouyang et al. (2015) considered the model with capacity constraint and order size dependent trade credit. Uthayakumar and Priyan (2013) derived two echelon inventory models with controllable setup cost and lead time under service level constraint with permissible delay in payment. Moreover, in the model developed by Uthayakumar and Priyan (2013), have considered two echelon inventory systems with delay in payment and controllable setup cost under service level constraint. In the present work, we have developed a single vendor single buyer integrated inventory model with permissible delay in payment and controllable lead time in an imperfect production process through service level constraint. An inspection policy is taken to identify the defective items. The lead time crashing cost is assumed to be an exponential function of lead time. The lead time demand follows a normal distribution and the lead time is crashed to minimize the joint total expected cost per unit time. The vendor's setup cost is reduced by some capital investment. Numerical example is provided to illustrate the model. Sensitivity analyses with managerial implications are discussed.

The rest of the paper is organized as follows. The assumptions and notations which are used throughout the article are presented in Section 2. In Section 3, mathematical model to minimize the total cost is formulated. In Section 4, the solution methodology comprising some useful theoretical result to find the optimal solution is given. Computational algorithm is designed to obtain the optimal values in the Section 5. Numerical example is provided in Section 6 to illustrate the theory and the solution procedure. Following this, in Section 7, Sensitivity analysis for the major parameters of the inventory system has been analyzed. Managerial implications with respect to the sensitivity analysis were given in Section 8. Finally, we draw a conclusion in Section 9.

## 2. NOTATIONS AND ASSUMPTIONS

### 2.1 Notations

The following notations are used throughout this article.

| | |
|---|---|
| $D$ | Buyer's annual demand rate in units per unit time |
| $P$ | Vendor's production rate in units per unit time, $P > D$ |
| $k$ | Buyer's ordering cost per order |
| $\varepsilon$ | Probability that an item produced is defective |
| $f(\varepsilon)$ | The probability density function of $\varepsilon$ |
| $h_b$ | Buyer's holding cost rate per unit per unit time |
| $h_v$ | Vendor's holding cost rate per unit per unit time |
| $F$ | The freight (transportation) cost per shipment from the vendor to the buyer |
| $M$ | The length of the trade credit period, in years |
| $I_p$ | Interest charge to be paid per \$ per year |
| $I_e$ | Rate of Interest earned for the buyer \$ per year |
| $I_v$ | Rate of Interest for calculating vendor's opportunity interest loss due to the delay payment, \$ per year |
| $c_b$ | Unit purchase cost paid by the buyer |
| $p$ | Unit selling price for the buyer, $c_b < p$ |
| $r$ | Reorder point of the buyer |
| $B(r)$ | Expected demand shortage at the end of the cycle |
| $X$ | The lead time demand in units per unit time, a random variable |

**Decision Variables**

$Q$ Buyer's order quantity in units

$L$ The length of lead time for the buyer

$k_v$ Setup cost per production run for the vendor

$n$ The number of lots in which the product is delivered from the vendor to the buyer in one production cycle, a positive integer

### 2.2 Assumptions

1. The inventory model deals with single-buyer and single-vendor with single item.
2. The buyer orders a lot size of $Q$ units and the vendor produces $nQ$ units with a finite production rate $P$, $(P > D)$ in units per unit time in one setup but ships in quantity $Q$ units to the buyer over $n$ times. The buyer's shortages are completely backordered.
3. The production process is imperfect and may produce defective items. On arrival, the items are inspected in a complete inspection process with an inspection cost of $c_i$ and all defective items are returned to the vendor in the next shipment. A defective item incurs a cost of $c_d$ for the vendor. The vendor will sell the defective items at a reduced price to a secondary market at the end of the production period within each cycle. In other words, $c_d$ is the difference between the regular and the reduced selling prices.
4. The percentage of defective items produced $\varepsilon$ has a probability density function $f(\varepsilon)$. To guarantee that the vendor has enough production capacity to produce the buyer's annual demand, it is assumed that $\varepsilon < 1 - D/P$.
5. The inventory is continuously reviewed and the order is placed whenever the inventory level falls to the reorder point $r$. The reorder point $r = DL + \tau\sigma\sqrt{L}$, where $DL$ is the expected demand during lead time, $\tau$ the safety factor and satisfies $Pr(X > r) = q$, $q$ represents the allowable stock-out probability during lead time and $\sigma$, the standard deviation of the lead time demand.
6. The lead time crashing cost per order $R(L)$, is assumed to be an exponential function of $L$ and is defined as $R(L) = \begin{cases} 0 & if\ L = L_0 \\ e^{C/L} & if\ L_b \le L < L_0 \end{cases}$, where $C$ is a positive constant and $L_0$ and $L_b$ represent the existing and the shortest lead times, respectively.
7. $M$ is less than the reorder point, i.e. credit period should not be longer than the time at which next order is placed.

## 3. MODEL DEVELOPMENT

In this section, based on the above notations and assumptions, we have developed a model for vendor-buyer integrated inventory system for defective items, with controllable lead time and setup cost under permissible delay in payment to minimize the joint total expected cost per unit time subject to service level constraint on the buyer. The inventory pattern for the vendor and the buyer is shown in Figure 1.

### 3.1 Buyer's expected total cost

Buyers total expected cost consists of the following components:

1. The Buyer's ordering cost per unit time is $\frac{kD}{Q}$.
2. Inventory holding cost per unit time is $\frac{c_b\, h_b Q}{2}$.
3. The total safety stock cost per unit time is the sum of the holding cost and interest charged
4. i.e. $(h_b + I_p)c_b\tau\sigma\sqrt{L}$.
5. According to our assumption, the credit period cannot be greater than the ordering time. Therefore, when the buyer's permissible delay period expires on or before all inventories are depleted completely, the buyer can sell the items and earn interest with the rate of $I_e$ until the end of the credit period $M$. Hence, the buyer's interest earned per unit time is $\frac{D^2M^2pI_e}{2Q}$.
6. In addition, the expected shortage $B(r)$ is completely backordered in the previous cycle, which is cleared in the beginning of the current cycle, therefore, during the trade credit period the buyer earns an interest of $\frac{DMpI_eB(r)}{Q}$ per unit time. On the other hand, the buyer still has $(Q - DM)$ units unsold at the end of the permissible delay. Hence, the buyer has a loan for the unpaid purchase cost of unsold units with the interest charge of $I_p$. Therefore, the opportunity interest cost per cycle time for the unsold items is obtained by $\frac{(Q-DM)^2c_b\, I_p}{2Q}$.
7. The lead time crashing cost per unit time is given by $\frac{DR(L)}{Q}$.

Hence the total expected cost per unit time for the buyer consists of ordering cost, holding cost, safety stock cost, opportunity interest cost, lead time crashing cost, interest earned and the transportation cost is expressed as

$$EAC_b(Q,L) = \frac{kD}{Q} + \frac{Qc_bh_b}{2} + \left(h_b + I_p\right)c_b\tau\sigma\sqrt{L} + \frac{(Q-DM)^2c_b\,I_p}{2Q} + \frac{DR(L)}{Q} - \frac{D^2M^2pI_e}{2Q} - \frac{DMpI_eB(r)}{Q} + \frac{FD}{Q} \tag{1}$$

### 3.2 Vendor’s expected total cost

Vendor's total expected cost consists of the following components.

1. The set-up cost is given by $k_v$.
2. The holding cost for the defective items and good quality are $h_v\left\{\left(\frac{nQ}{2}\left[\frac{\varepsilon}{1-\varepsilon}\right]\right)\left(\frac{nQ}{P}\left[\frac{1}{1-\varepsilon}\right]\right)\right\}$and $h_v\left\{\frac{nQ^2}{P(1-\varepsilon)}\left[1-\frac{n}{2}\right]+\frac{n(n-1)Q^2}{2D}\right\}$ respectively.
3. The cost of defective items is $\frac{c_dnQ\varepsilon}{(1-\varepsilon)}$.
4. Inspection cost = $\frac{c_inQ}{(1-\varepsilon)}$
5. The vendor’s opportunity interest loss per unit time is $I_vc_bMnQ$.

Hence, the vendor’s total expected cost per unit time compresses, the setup cost, inspection cost, the cost of defective items, holding cost and opportunity interest loss. Therefore

$$EAC_v(Q,n) = \frac{k_vD}{nQ} + c_iDE\left[\frac{1}{1-\varepsilon}\right] + c_dDE\left[\frac{\varepsilon}{1-\varepsilon}\right] + h_v\left\{\frac{QD}{P}\left[1-\frac{n}{2}\right]E\left[\frac{1}{1-\varepsilon}\right] + \frac{(n-1)Q}{2} + \frac{nQD}{2P}E\left[\frac{\varepsilon}{(1-\varepsilon)^2}\right]\right\} + I_vc_bMD \tag{2}$$

### 3.3 The integrated vendor-buyer inventory model

The total cost per unit time for the vendor-buyer integrated inventory system under permissible delay in payment is given by

$$EAC(Q,L,n) = EAC_b(Q,L) + EAC_v(Q,n)$$

$$\begin{aligned} EAC(Q,L,n) = {} & \frac{kD}{Q} + \frac{Qc_bh_b}{2} + \left(h_b + I_p\right)c_b\tau\sigma\sqrt{L} + \frac{(Q-DM)^2c_b\,I_p}{2Q} + \frac{DR(L)}{Q} - \frac{D^2M^2pI_e}{2Q} - \frac{DMpI_eB(r)}{Q} \\ & + \frac{FD}{Q} + \frac{k_vD}{nQ} + c_iDE\left[\frac{1}{1-\varepsilon}\right] + c_dDE\left[\frac{\varepsilon}{1-\varepsilon}\right] \\ & + h_v\left\{\frac{QD}{P}\left[1-\frac{n}{2}\right]E\left[\frac{1}{1-\varepsilon}\right] + \frac{(n-1)Q}{2} + \frac{nQD}{2P}E\left[\frac{\varepsilon}{(1-\varepsilon)^2}\right]\right\} + I_vc_bMD \end{aligned} \tag{3}$$

where $E[x]$ denotes the expected value of $x$.

### 3.4 Buyer’s Service level constraint

The lead time demand $X$ follows a normal probability distribution function with mean $DL$ and standard deviation $\sigma\sqrt{L}$and the reorder point, $r = DL + \tau\sigma\sqrt{L}$, where $\tau$ is the safety factor and $\sigma$, the standard deviation of the lead time demand. Therefore, the buyer's expected demand shortages at the end of the cycle is given by

$$E(X-r)^+ = B(r) = \int_r^\infty (x-r)f(x)dx = \sigma\sqrt{L}\,\psi(\tau)$$

where $\psi(\tau) = \varphi(\tau) - \tau[1-\phi(\tau)] > 0$ and $\varphi, \phi$ are the standard normal p.d.f and cumulative distribution function respectively. In developing, the joint total expected cost of the system, the stockout cost term for the buyer is not considered. Normally, it is complicated to calculate the penalty costs associated with a shortage, as a stock-out event may include uncertain manipulates. As a result, authors like Jha and Shankar (2009, 2013), Moon et. al. (2014), Annadurai and Uthayakumar (2010) assumed that the buyer has set a target service level corresponding to the proportion of demand to be satisfied directly from the available stock. Hence, a service level constraint puts a limit on the proportion of demand that is not met from the stock. From assumption (5), the actual proportion of demand that is not met from the stock should not exceed the desired value of $\alpha$. Hence, the Service level constraint can be expressed as

$$\frac{\text{Expected demand shortages at the end of the cycle for a given safety factor}}{\text{Quantity available for satisfying the demand per cycle}} \le \alpha$$

i.e. ,

$$\frac{B(r)}{Q} \le \alpha$$

When the lead time demand follows normal distribution, Service level constraint is given by

$$\frac{\sigma\sqrt{L}\,\psi(\tau)}{Q} \le \alpha$$

Therefore, the joint total expected cost per unit time for the vendor-buyer integrated inventory system under permissible delay in payment is given by

$$\begin{aligned} &EAC(Q, L, k_v, n) \\ &= \frac{D}{Q}\left\{k + \frac{k_v}{n} + R(L) - MpI_e\sigma\sqrt{L}\,\psi(\tau) + F + \frac{DM^2}{2}\left(c_bI_p - pI_e\right)\right\} \\ &+ \frac{Q}{2}\left[\left(h_b + I_p\right)c_b + h_v\left\{\frac{D}{P}[2-n]E\left[\frac{1}{1-\varepsilon}\right] + (n-1) + \frac{nD}{P}E\left[\frac{\varepsilon}{(1-\varepsilon)^2}\right]\right\}\right] \\ &+ \left(h_b + I_p\right)c_b\tau\sigma\sqrt{L} + c_iDE\left[\frac{1}{1-\varepsilon}\right] + c_dDE\left[\frac{\varepsilon}{1-\varepsilon}\right] + DMc_b\left(I_v - I_p\right) \end{aligned}$$

subject to

$$\frac{\sigma\sqrt{L}\,\psi(\tau)}{Q} \le \alpha \tag{4}$$

### 3.5 Investment in setup cost reduction

In this section, the effects of investments on setup cost reduction is studied. Here we have considered the setup cost $k_v$ as a decision variable and have sought seek to minimize the sum of the capital investment cost and the total cost of the integrated inventory system. In real life, the setup cost can be controlled and reduced through efforts such as procedural changes, worker training and specialized acquisition. According to Nasri et al. (1990), the implementation of electronic data interchange may reduce the fixed setup cost and result in new replenishment policy and the corresponding lower cost. Many researchers such as Porteus (1985), Paknejad and Affisco (1987), Kim et al. (1992), Hall (1983), Uthayakumar and Priyan (2013) discussed several classes of setup cost reduction functions. In this article, we have assumed that the relationship between the setup cost reduction and capital investment by the logarithmic investment function. In many business transactions, the logarithmic function is used to determine the present and the future value of investments.

Hence, the logarithmic investment function discussed here is not only an interesting special case but also a practical one. Specifically for the logarithmic investment functions the setup cost $k_v$ declines exponentially as the investment amount $I_{k_v}$ increases, i.e., (following Nasri et al. (1990))

$k_v = k_{v_0}\exp(-\delta I_{k_v})$ for $0 \le I_{k_v} < \infty$

where $k_{v0}$ is the original setup cost and $\delta$ is the percentage decrease in $k_v$ per dollar increase in $I_{k_v}$. Taking the natural logarithm of both sides of the above equation yields,

$$I_{k_v}(k_v) = A - B\,ln(k_v) \text{ for } 0 < k_v \le k_{v_0}$$

where $A = \frac{ln(k_{v_0})}{\delta}$ and $B = \frac{1}{\delta}$

Hence, the logarithmic investment function is stated as

$$I_{k_v}(k_v) = B\,ln\left(\frac{k_{v_0}}{k_v}\right) \, for\ 0 < k_v \le k_{v_0}$$

where $\frac{1}{B}$ is the fraction of the reduction in $k_v$ per dollar increase in investment. Thus the annual cost of such an investment is $\xi\, I_{k_v}(k_v)$, where $\xi$, is the annual fractional cost of capital investment.

Now, the problem is to find the optimal lot size, lead time, setup cost and the total number of deliveries in a production cycle that minimize the joint total expected cost, that is,

$$\begin{aligned} &\min EAC\,(Q, L, k_v, n) \\ &= \xi\, I_{k_v}(k_v) + \frac{D}{Q}\left\{k + \frac{k_v}{n} + R(L) - MpI_e\sigma\sqrt{L}\,\psi(\tau) + F + \frac{DM^2}{2}\left(c_bI_p - pI_e\right)\right\} \\ &+ \frac{Q}{2}\left[\left(h_b + I_p\right)c_b + h_v\left\{\frac{D}{P}[2-n]E\left[\frac{1}{1-\varepsilon}\right] + (n-1) + \frac{nD}{P}E\left[\frac{\varepsilon}{(1-\varepsilon)^2}\right]\right\}\right] \\ &+ \left(h_b + I_p\right)c_b\tau\sigma\sqrt{L} + c_iDE\left[\frac{1}{1-\varepsilon}\right] + c_dDE\left[\frac{\varepsilon}{1-\varepsilon}\right] \\ &+ DMc_b\left(I_v - I_p\right) \end{aligned}$$

subject to
$$0 < k_v < k_{v0}$$
$$\frac{\sigma\sqrt{L}\,\psi(\tau)}{Q} \leq \alpha \qquad (5)$$

## 4. SOLUTION PROCEDURE

In order to find the minimum total cost for this constrained nonlinear programming problem, we first have temporarily ignored the service level constraint (SLC), $0 < k_v < k_{v0}$ and have relaxed the integer requirement on $n$, then have tried to find the optimal solution of $EAC(Q, L, k_v, n)$with classical optimization technique.

For fixed $Q$ and $L \in (L_b, L_0)$, $EAC(Q, L, k_v, n)$ can be proved to be a convex function of $n$.

**Proposition 1:**

*For fixed* $Q, k_v$ *and* $L \in (L_b, L_0)$, $EAC(Q, L, k_v, n)$ *is convex in* $n$.

***Proof:*** Taking the first and the second partial derivatives of $EAC(Q, L, n)$ with respect $n$, we have

$$\frac{\partial EAC(Q, L, k_v, n)}{\partial n} = -\frac{Dk_v}{Qn^2} + \frac{Qh_v}{2}\left\{\frac{-D}{P}E\left[\frac{1}{1-\varepsilon}\right] + 1 + \frac{D}{P}E\left[\frac{\varepsilon}{(1-\varepsilon)^2}\right]\right\}$$

$$\frac{\partial^2 EAC(Q, L, k_v, n)}{\partial n^2} = \frac{2Dk_v}{Qn^3} > 0.$$

Therefore, for fixed $Q, k_v$ and $L \in (L_b, L_0)$,$EAC(Q, L, k_v, n)$ is convex in $n$.

Now, for fixed $n$ we take the first order partial derivatives of $EAC(Q, L, k_v, n)$ with respect to $Q, k_v$ *and* $L \in (L_b, L_0)$ respectively, we obtain

$$\frac{\partial EAC(Q, L,\ k_v, n)}{\partial Q} = -\frac{D}{Q^2}\left[k + \frac{k_v}{n} + R(L) - MpI_e\sigma\sqrt{L}\,\psi(\tau) + F + \frac{DM^2}{2}(c_bI_p - pI_e)\right] + \frac{1}{2}\left[(h_b + I_p)c_b + h_v\left\{\frac{D}{P}[2-n]E\left[\frac{1}{1-\varepsilon}\right] + (n-1) + \frac{nD}{P}E\left[\frac{\varepsilon}{(1-\varepsilon)^2}\right]\right\}\right] \qquad (6)$$

and

$$\frac{\partial EAC(Q, L,\ k_v, n)}{\partial\ k_v} = \frac{D}{nQ} - \frac{\xi B}{k_v} \qquad (7)$$

By setting the above equations (6) and (7) to zero, we obtain

$$Q = \left\{\frac{2D\left[k + \frac{k_v}{n} + R(L) - MpI_e\sigma\sqrt{L}\,\psi(\tau) + F + \frac{DM^2}{2}(c_bI_p - pI_e)\right]}{(h_b + I_p)c_b + h_v\left\{\frac{D}{P}[2-n]E\left[\frac{1}{1-\varepsilon}\right] + (n-1) + \frac{nD}{P}E\left[\frac{\varepsilon}{(1-\varepsilon)^2}\right]\right\}}\right\}^{\frac{1}{2}} \qquad (8)$$

and

$$k_v = \frac{\xi BnQ}{D} \qquad (9)$$

Hence, for fixed $n$ and $L \in (L_b, L_0)$, we can obtain the optimal values of $Q$ & $k_v$.

## 5. COMPUTATIONAL ALGORITHM

Step 1: Set $n = 1$.

Step 2: For each $L \in [L_b, L_0]$ perform (2.1–2.4).

2.1 Start with $k_{v1} = k_{v0}$.

2.2 Substituting $k_{v1}$ into equation (8) evaluate $Q_1$.

2.3 Utilizing $Q_1$ determine $k_{v2}$ from equation (9)

2.4 Repeat (2.2–2.3) until no change occurs in the values of $Q$ and $k_v$. Denote the solution by$\left(\dot{Q},\ \dot{k_v}\right)$.

Step 3: Compare $\dot{k_v}$ with $k_{v0}$.

3.1 If $\dot{k_v} < k_{v0}$ then go to step 4.

3.2 If $\dot{k_v} > k_{v0}$, then for $L \in [L_b, L_0]$, let $\dot{k_v} = k_{v0}$ and utilize equation (8) (replace $k_v$by $k_{v0}$), to determine the new $\dot{Q}$, then go to step 4.

Step 4: Let $\bar{Q} = max\{\dot{Q},\ (\sigma/\alpha)\sqrt{L}\,\psi(\tau)\}$ then determine the new $\dot{k_v}$(the result is denoted as $\overline{k_v}$) by putting $Q = \bar{Q}$ in equation (9) and perform steps 4.1 and 4.2.

4.1 If $\overline{k_v} < k_{v0}$, then go to step 5.

4.2 If $\overline{k_v} > k_{v0}$, then for this $L \in [L_b, L_0]$, let $\overline{k_v} = k_{v0}$ then go to step 5.

Step 5: Compute the corresponding $EAC\left(\bar{Q},\overline{k_v}, L, n\right)$by putting $Q = \bar{Q}$ and $k_v = \overline{k_v}$ in equation (5).

Step 6: Find $\min_{L\in[L_b,L_0]} EAC\left(\bar{Q},\overline{k_v}, L, n\right)$. Let

$EAC\left(Q_{(n)}{}^*,\ k_{v(n)}{}^*, L_{(n)}{}^*, n\right) =$
$\min_{L\in[L_b,L_0]} EAC\left(\bar{Q},\overline{k_v}, L, n\right)$,

then$\left(Q_{(n)}{}^*,\ k_{v(n)}{}^*, L_{(n)}{}^*, n\right)$ is the optimal solution for fixed $n$.

Step 7: Set $n = n + 1$, repeat steps 2-6 to get $EAC\left(Q_{(n)}{}^*,\ k_{v(n)}{}^*, L_{(n)}{}^*, n\right)$.

Step 8: If

$EAC\left(Q_{(n)}{}^*,\ k_{v(n)}{}^*, L_{(n)}{}^*, n\right) \le$
$EAC\left(Q_{(n-1)}{}^*,\ k_{v(n-1)}{}^*, L_{(n-1)}{}^*, n-1\right)$,

then go to step 7, otherwise go to step 9.

Step 9: Set

$\left(Q_{(n)}{}^*,\ k_{v(n)}{}^*, L_{(n)}{}^*, n^*\right) =$
$\left(Q_{(n-1)}{}^*,\ k_{v(n-1)}{}^*, L_{(n-1)}{}^*, n-1\right)$,

then $EAC\left(Q_{(n)}{}^*,\ k_{v(n)}{}^*, L_{(n)}{}^*, n^*\right)$ is the optimal solution.

## 6. NUMERICAL EXAMPLE

In order to illustrate the above solution procedure, let us consider an inventory system with the following characteristics:

$D$ = 1000 units/year, $P$ = 1500 units/year, $k$ = \$200/order, $k_{v_0}$ = \$400/setup, $h_b$= \$4/unit/year, $h_v$= \$2/unit/year, $F$ = \$25/delivery, $c_b$= \$25/unit, $p$ = \$30/unit, $\tau$ = 0.75 (the value of $\psi(\tau)$ can be found directly from the standard normal table and is 0.1311), $\sigma$ = 7 units per week, where 1 year = 52 weeks, the service level $(1-\alpha)$ = 0.985, $c_i$= \$0.5/unit, $c_d$= \$30/unit, $M$= 0.166year, $I_v$= 0.02\$/year, $I_e$=0.02\$/year, $I_p$= 0.06 \$/year, $B$= 800, $\xi$=0.1/dollar/year and the crashing cost

$R(L) = \begin{cases} 0 & if\ L = 10 \\ e^{C/L} & if\ 1 \le L < 10 \end{cases}$ where $C$= 5.

If the defective percentage follows a uniform distribution with $f(\varepsilon) = \begin{cases} \frac{1}{\beta}, 0 \le \epsilon \le \beta \\ 0, otherwise \end{cases}$

then

$$E\left[\frac{1}{1-\varepsilon}\right] = \int_0^\beta \frac{1}{1-\varepsilon} f(\varepsilon)d\varepsilon = -\frac{\ln(1-\beta)}{\beta}.$$

$$E\left[\frac{\varepsilon}{1-\varepsilon}\right] = E\left[\frac{1}{1-\varepsilon} - 1\right] = E\left[\frac{1}{1-\varepsilon}\right] - 1 = -\frac{\ln(1-\beta)}{\beta} - 1.$$

$$E\left[\frac{1}{(1-\varepsilon)^2}\right] = \frac{1}{1-\beta},\ \ and$$

$$E\left[\frac{\varepsilon}{(1-\varepsilon)^2}\right] = E\left[\frac{1}{(1-\varepsilon)^2} - \frac{1}{1-\varepsilon}\right] = \frac{1}{1-\beta} + \frac{\ln(1-\beta)}{\beta}.$$

Applying the proposed computational algorithm, the optimal values (when$\beta = 0.02$ ) are

$L^* = 2$ weeks,

$Q^* = 129.37$ units,

$k_v{}^* = \$62.10/\text{setup}$,

number of deliveries $n^* = 6$ and

the corresponding minimum integrated optimal total expected cost $EAC(Q^*, L^*,\ k_v{}^*, n^*) = \$3976.80$.

## 7. SENSITIVITY ANALYSIS

Now, we examine the effects of changes in the system parameters $k$, $k_v$, $D$, $\beta$ and $M$ on the optimal ordering quantity $Q$ and setup cost $k_v$ and total expected annual cost. The optimal values of $Q, L, k_v, n$ and $EAC(Q, L, k_v, n)$ are obtained when one of the parameters changes (increases or decreases) by 25% and all the other parameters remain unchanged. The results of sensitivity analysis are presented in Table 1. On the basis of the results obtained, the following observations can be made:

1. From Table-1, we infer that the demand rate $D$ is sensitive to its expected total cost. $Q^*$ and the

integrated expected total cost $EAC(Q^*, L^*, k_v{}^*, n^*)$ increase while $k_v{}^*$ decreases with an increase in the value of the parameter $D$. The results obtained are given in Fig 2.

2. When the ordering cost $k$ increases, the optimal order quantity $Q^*$ and the total expected cost $EAC(Q^*, L^*, k_v{}^*, n^*)$ increases without affecting the lead time. The graphical representation is shown in Fig 3.
3. From Fig 4, we see that, when the holding cost $h$ increases, there is an increase in the optimal order quantity $Q^*$ and the total expected cost $EAC(Q^*, L^*, k_v{}^*, n^*)$.
4. From Fig 5, we see that, increase in $\beta$ , results in increase in the expected total cost $EAC(Q^*, L^*, k_v{}^*, n^*)$of the integrated system.
5. When duration of the credit period $M$ increases, the expected total cost $EAC(Q^*, L^*, k_v{}^*, n^*)$ of the integrated system decreases. The graphical representation is shown in Fig 6.

## 8. MANAGERIAL IMPLICATIONS

In this section, we present some managerial insights of the proposed model based on the numerical results and sensitivity analyses. When the demand rate is high, the vendor may lose production efficiency which results in high total cost. Therefore, the design of production capacity is important in controlling the production cost. Also, the benefit of the integrated model is more significant for high values of demand rate. From inventory point of view, the retailer should order more quantity per order when the ordering cost is high. Also, it is advised that the retailer should take steps to reduce the ordering cost per order by some capital investment. The obtained result shows that the integrated expected total cost of our system is sensitive with increase in $h$. From Fig 5, we infer that, increase in $\beta$ , results in increase in the expected total cost of the integrated system. For higher values of $\beta$, the total expected cost is high and the order quantity is low. From the managerial view point, it is advised that the supplier should find some measure to decrease the defective rate. That is, the supplier should entertain quality production to reduce the total expected cost of the supply chain. From economical point of view, if the supplier provides a permissible delay in payments, the retailer will order lower quantity in order to take the benefits of the permissible delay more frequently.

## 9. CONCLUSION AND FUTURE WORK

In this article, a mathematical model is developed to determine an optimum integrated vendor-buyer inventory policy for defective items where permissible delay in payment is offered by the vendor to the buyer. The vendor's setup cost is crashed by an extra crashing cost. Lead time is considered and the lead time demand follows a normal distribution. The lead time, order quantity, setup cost and the number of shipments per production cycle are obtained considering the service level constraint so that, the expected total system cost can be minimized. By controlling the setup cost by an investment, significant amount of savings can be achieved. Our results show that, when the vendor offers trade credit period, the buyer should use it to the maximum extent and order more quantity so that the expected total cost of the integrated system is reduced. From our analysis, when the defective rate is high, the buyer should order less, i.e., the quality of the product plays a vital role in the integrated model and necessary steps may be taken by the vendor to reduce the defective rate. The proposed model can be used in industries like refrigerator, air-conditioner, washing machine, television, printers etc.

In future, possible extension of this work may be conducted by considering the minimax distribution free procedure to determine the optimal values and also it would be interesting research topic to consider general types of investment functions and their associated marginal cost behavior. This model can be further extended by considering the reorder point as a decision variable.

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Table 1. Sensitivity analysis with respect to the major parameters

| Parameter | % change | $n$ | $L$ | $k_v$ | $Q$ | $EAC(Q^*, L^*, k_v{}^*, n^*)$ |
|---|---|---|---|---|---|---|
| $D$ | +50% | 7 | 2 | 67.23 | 186.77 | 5877.10 |
| | +25% | 7 | 2 | 82.34 | 123.71 | 4920.30 |
| | -25% | 6 | 2 | 98.16 | 104.86 | 3230.23 |
| | -50% | 6 | 2 | 101.45 | 90.58 | 2750.10 |
| $k$ | +50% | 6 | 2 | 69.56 | 149.11 | 4012.80 |
| | +25% | 6 | 2 | 90.81 | 133.58 | 3981.76 |
| | -25% | 6 | 2 | 102.30 | 119.86 | 3101.92 |
| | -50% | 6 | 2 | 112.59 | 102.31 | 2633.10 |
| $\beta$ | 0.04 | 6 | 2 | 87.12 | 119.03 | 3013.56 |
| | 0.06 | 6 | 2 | 80.56 | 101.45 | 3470.91 |
| | 0.08 | 6 | 2 | 71.63 | 95.63 | 3601.78 |
| | 0.10 | 6 | 2 | 56.01 | 87.52 | 3846.12 |
| $M$ | +50% | 6 | 2 | 58.76 | 141.02 | 3763.45 |
| | +25% | 6 | 2 | 63.18 | 134.53 | 3870.93 |
| | -25% | 6 | 2 | 76.03 | 122.15 | 4002.86 |
| | -50% | 6 | 2 | 88.43 | 120.63 | 4131.12 |
| $h$ | +50% | 6 | 2 | 86.11 | 155.12 | 5021.87 |
| | +25% | 6 | 2 | 103.45 | 136.40 | 4783.03 |
| | -25% | 6 | 2 | 119.61 | 119.81 | 3986.41 |
| | -50% | 6 | 2 | 123.46 | 107.09 | 3401.30 |

Figure 1: The inventory pattern for the vendor and the buyer

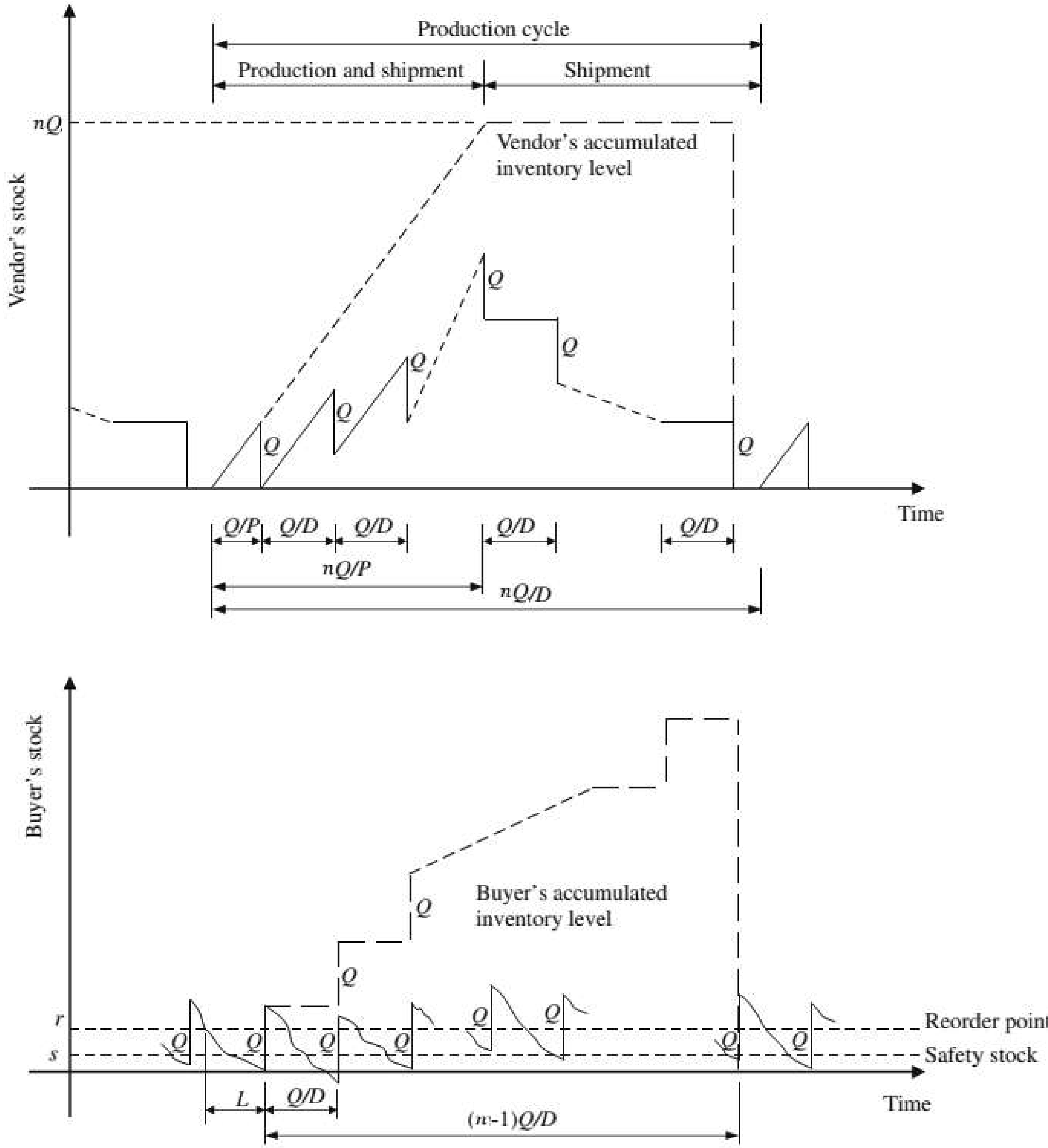


Figure 2: Effect of change in $D$ on the optimal solution.

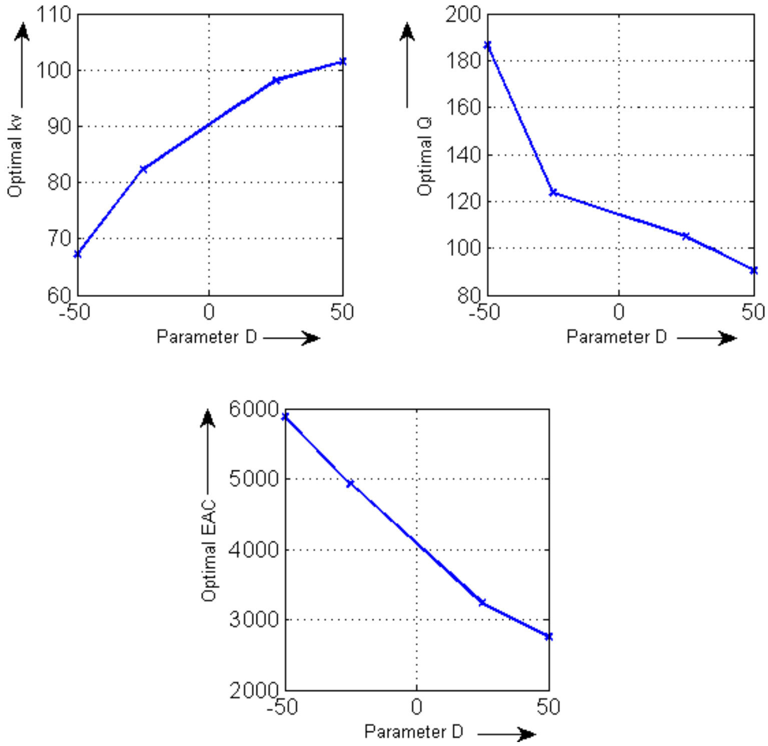



Figure 3: Effect of change in $k$ on the optimal solution.

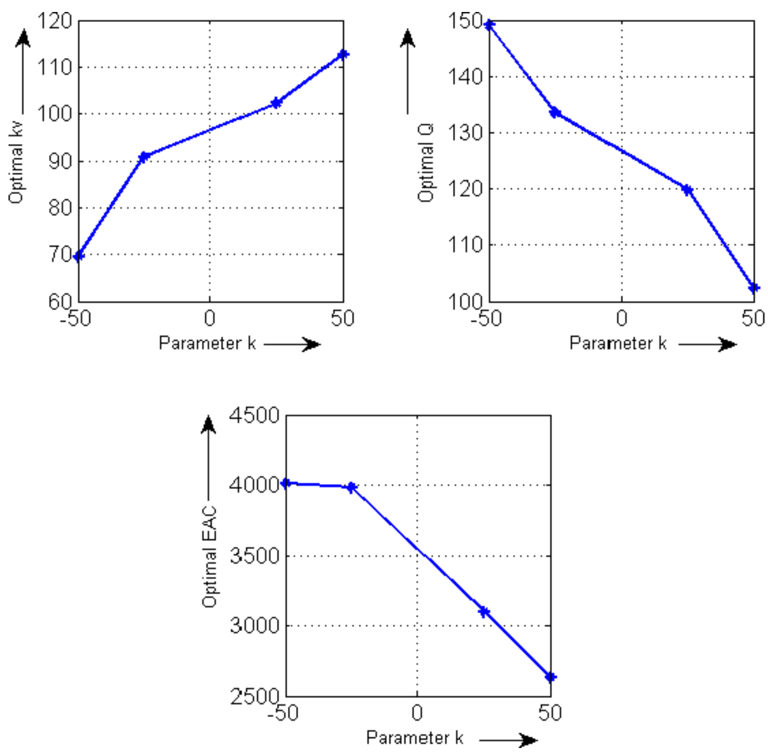


Figure 4: Effect of change in $h$ on the optimal solution

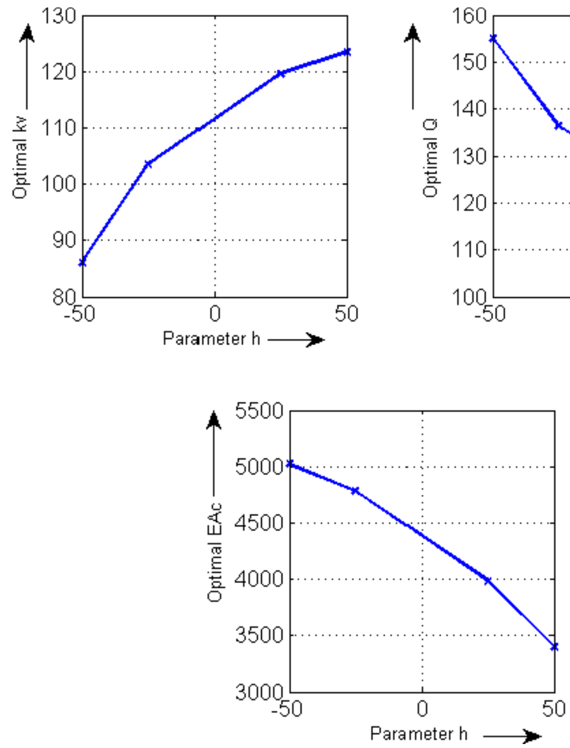



Figure 5: Effect of change in $\beta$ on the optimal solution

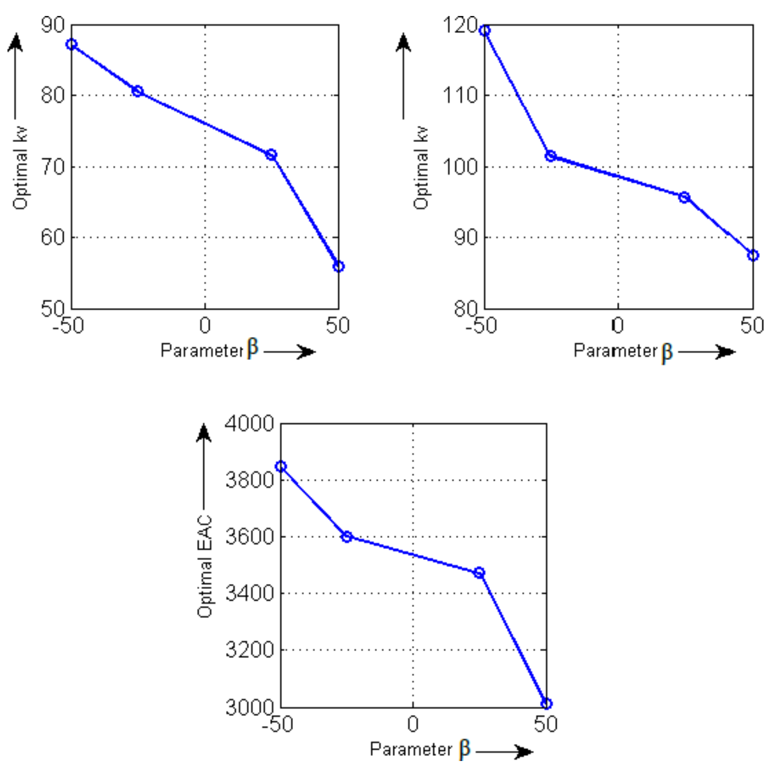
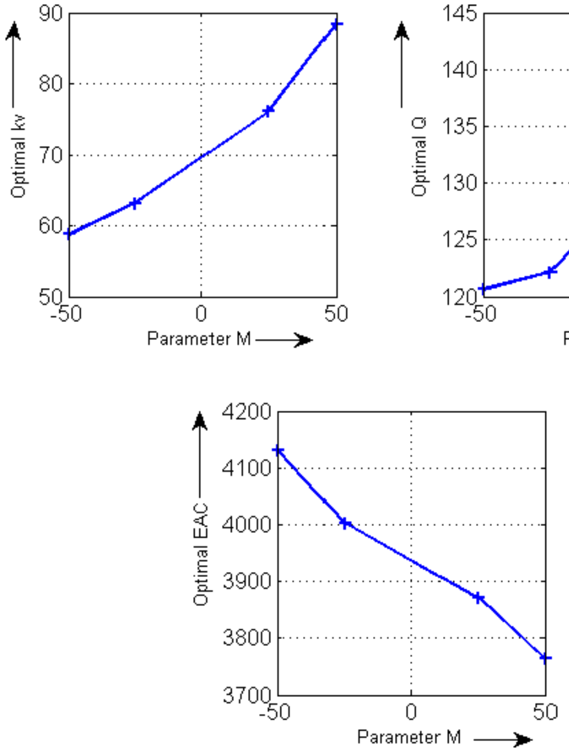


Figure 6: Effect of change in $M$ on the optimal solution.

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