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REVIEW ARTICLE

A review on stochastic models and analysis of warehouse operations

Yeming Gong · René B. M. de Koster

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Abstract This paper provides an overview of stochastic research in warehouse operations. We identify uncertainty sources of warehousing systems and systematically present typical warehouse operations from a stochastic system viewpoint. Stochastic modeling methods and analysis techniques in existing literature are summarized, along with current research limitations. Through a comparison between potential and existing stochastic warehouse applications, we identify potential new research applications. Furthermore, by comparing potential and existing solution methods, methodological directions relevant to practice and largely unexplored in warehouse literature are identified.

Keywords Facilities planning and design · Stochastic models · Stochastic optimization · Warehouse systems

1 Introduction

To stay competitive in a dynamic business environment full of uncertainties, today's warehouse operations face challenges like the need for shorter lead times, for real-time response, to handle a larger number of orders with greater variety, and to deal with flexible processes with a far greater complexity. Some online retailers, for example, face customers who purchase by impulse, then change their minds and legally cancel orders. Warehouses of these online retailers face uncertainty from real-time order information updates (see Gong and De Koster [1]). Therefore, warehouse managers must consider uncertainties from various sources, both from the outside supply chain and from within the warehouse itself. These uncertainties may come from unpredictable rare events, predictable trends, and internal variability of supply chain processes. Each of the uncertainty sources may cause an unanticipated impact on strategic, tactical, or operational decisions, yet must be met on a daily basis in practice.

Typical warehouse operations include receiving, putaway, internal replenishment, order picking, accumulating, sorting, packing, cross docking, and shipping. Internal variability of these warehouse processes can be observed in the variability of, for example, the putaway quantity, the replenishment quantity and the cross-docking quantities, but also in a variety of other forms. In order picking, for example, the pick route length is variable and batch sizes vary as well. Also, product or shipment queuing in sorting systems induces variability. Over the last decades, warehouses have had to learn to adapt to an increasing amount of uncertainty. Many warehouses have attempted innovative approaches to order receiving, storage, order picking, and shipping to mitigate risks and to handle the challenge from uncertainties. This has also led to a far greater complexity in warehouse operations [2] and the use of new warehouse systems like dynamic storage, real-time processing, and dynamic picking (see De Koster et al. [3]), where products do not have fixed slots, orders are released dynamically, and the travel time of pickers is reduced.

Deterministic models and algorithms are successful in the research of warehouse systems (e.g., Ratliff and Rosenthal [4], Van den Berg et al. [5], Karasawa et al. [6],

Y. Gong (✉)
EMLYON Business School, 23 avenue Guy de Collongue, 69134 Ecully Cedex, France
e-mail: gong@em-lyon.com

R. B. M. de Koster
Rotterdam School of Management, Erasmus University, P.O. Box 1738, 3000 DR Rotterdam, The Netherlands
e-mail: rkoster@rsm.nl

Lowe et al. [7], White and Francis [8]). Even though real-world business problems always have some stochastic factors, deterministic models can provide a good approximation in a stable business setting. However, deterministic models may not always suffice in highly variable environments such as in systems with strongly fluctuating order patterns and responsive operations (e.g., online order handling). In fact, they even may lead to wrong conclusions if underlying processes are variable.

To handle problems with internal variability, a number of stochastic warehouse models have been developed (e.g., Bozer and White [9], De Koster [10], Chew and Tang [11], Bartholdi et al. [12]). These pioneering researches provide a valuable start for exploration in warehouse research by stochastic methods. One of our motivations is to provide an overview of existing stochastic research in warehousing and to identify potential application directions. On the other hand, stochastic models and theory have evidently developed in the last 20 years. Warehouse practitioners and researchers need suitable methods to research warehouse problems in a stochastic environment. Stochastic models may help understand the impact of stochastic factors on the operational processes and system performance. While stochastic models are potentially efficient tools for warehouse research, their application to warehouse research is limited. Therefore, another motivation is to bridge this gap by identifying potential stochastic methods for warehouse research.

Several literature reviews on warehousing research exist. Gu et al. [13] carry out a comprehensive review on warehouse research. Van den Berg and Zijm [14] present a classification of warehouse management problems. Other researchers focus on an aspect of warehouse research. De Koster et al. [3] review the order-picking problem in warehouses. Cormier and Gunn [15] have classified the warehouse models into three categories, namely throughput capacity models, storage capacity models, and warehouse design models. Our research takes a totally different view and provides insights into method issues in a stochastic setting by identifying the uncertainty sources of warehouse operations, presenting a systematic overview of the stochastic models and analysis of warehouse operations, and further presenting potential research directions.

In order to identify appropriate academic warehouse literature, we searched via "ABI/INFORM Global", "ScienceDirect", "ISI Web of Knowledge", "Informaworld" and "Google Scholar", using key words and their derivatives like "warehouse", "distribution center","order-picking", "storage", "order retrieval", "order receiving" and "order shipping", "sorting", "AS/RS (Automated Storage and Retrieval Systems)". We identified 645 articles and 42 books in English on warehousing from 1948 to May 2010 (for a comprehensive list before 2008, see http://www.roodbergen.com). Literature on subjects, such as automated guided vehicles (AGV), facility layout (other than directly applied to warehousing), facility location and inventory models, has not been included. As this paper focuses on warehouse operations, warehouse design is not included. By carefully reading abstract, introduction and conclusion parts, and checking the remaining parts for research methods used in these 645 articles and 42 books, we identify the research using stochastic methods on warehouse operations. One of the limitations is that a large number of non-English literature is not included in this literature review, although some non-English literature is highly valuable. Furthermore, we group these papers by modeling types and methods, by analysis types and methods, and by warehouse processes studied. For each group, we discuss representative papers to illustrate the application of a method. We choose such papers mainly by the criterion whether the research fully fits within the category (e.g. full adoption of a method rather than partial adoption).

The remainder of the paper is organized as follows. In the following section, we identify uncertainty sources for warehouse operations. A methodological review of stochastic models and analysis of warehouse systems is presented in Sect. 3. In Sect. 4, we focus on different stochastic approaches and their potential for application in a warehouse context. Based on the analysis in Sects. 3 and 4, some limitations of current research are identified in Sect. 5. By comparing potential research (partially in Sect. 1) and existing research (mainly in Sects. 3 and 4) and the analysis of Sect. 5, we can point out future research directions in Sect. 6. Section 7 concludes this paper.

2 Warehouse operations: a stochastic view

This section identifies uncertainty sources of a warehouse system at strategic, tactical and operational levels, and presents uncertainties of a warehouse system in the warehouse arrival, service, and departure processes, three main processes of a stochastic system. The analysis in this section explains the necessity to research warehouse systems by stochastic methods in uncertain business settings, identifies potential opportunities of warehouse research by stochastic models and analysis, and provides a foundation for the further analysis in subsequent sections.

2.1 Uncertainty sources of warehouse systems

Uncertainty sources faced by warehouse systems are quite diverse, both within and external to the warehouse systems (see Chopra and Sodhi [16]). We first present the classification of uncertainty sources and then study the influence of uncertainty sources on warehouse operations and decisions.

According to the location of uncertainty sources, we classify them as (1) sources outside the supply chain, (2) sources in the supply chain but outside the warehouse, (3) sources inside the warehouse, and (4) sources within warehouse control systems. We present this scope dimension on the horizontal axis of Fig. 1. According to the variance structure of uncertainties, we classify uncertainty sources as (1) unpredictable events like war, strikes, floods, and hurricanes, which usually are rare events, (2) predictable events like demand seasonality, and (3) internal variabilities like variance of order waiting time for batching, which could be caused by internal randomness. We present this classification dimension on the vertical axis of Fig. 1. The figure also shows typical examples of uncertainty sources for different types. Examples in Fig. 1 primarily distribute along the diagonal of the matrix. Outside uncertainty sources usually are more unpredictable and will often bring high variance to warehouse operations. On the other hand, inside uncertainty sources usually are more predictable and only bring low variance to warehouse operations.

Uncertainty sources can affect decisions at three levels, strategic, tactical, and operational, classified by the planning horizon. Strategic decisions have a long-run effect, tactical decisions have an effect over the medium term (monthly or quarterly), and operational decisions are made on a daily basis [17]. Decisions on warehouse automation level, layout, and systems have a strategic effect. Tactical decisions mainly include the storage, order picking, and shipping tactical plans. Warehouse operational decisions include daily order-picking planning, daily resource planning, and daily warehouse information system management. We further illustrate the impact of uncertainty sources at these decision levels.

2.1.1 Strategic uncertainty sources

Some system-wide uncertainty sources like natural disasters, war, and terrorism can impose a long-term impact on warehouse operations; for example, Hurricane Katrina has a long-run influence on distribution networks and warehouse operations in the USA. After Hurricane Katrina, the USA required that each state set up an emergency warehouse to rapidly and safely provide healthcare products like antibiotics, antivirals, and vaccines during disasters. The Mississippi State Department of Health discovered its existing emergency warehouse needed improvements to meet federal requirements, and they took many measures like implementing a new inventory management system (see fishbowlinventory.com). After emergency warehouses were set up, they have been successfully used at least once: they were activated in response to the H1N1 influenza outbreak in 2009. Other uncertainties, like those in facility and labor costs, in relation to facility productivity and labor productivity will influence the trade-off between operational capabilities and economic efficiency, and further influence strategic decisions on warehouse automation. Uncertainties in total ownership costs of costly resources

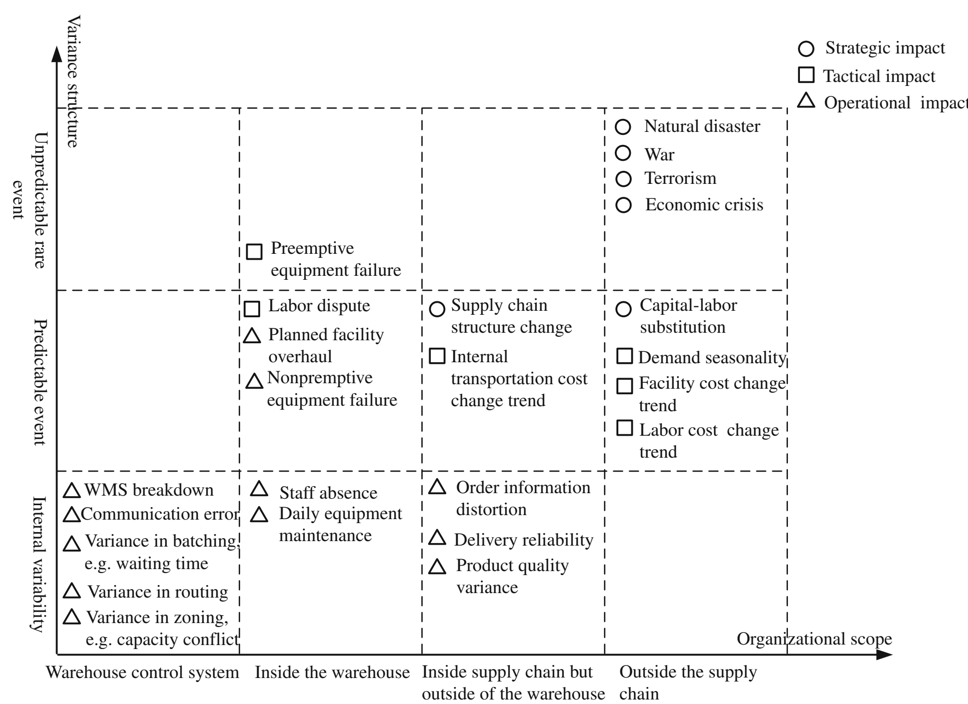

Fig. 1 Uncertainty sources of warehouse operations

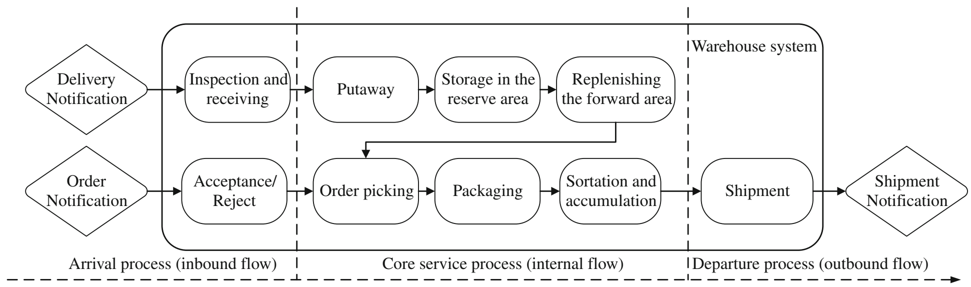

Fig. 2 Typical warehouse operations from a stochastic process view

(including staff, and key equipment like storage and sorting systems) may affect the financial performance of a warehouse over years.

2.1.2 *Tactical uncertainty sources*

Tactical uncertainty sources originate from both outside and inside the warehouse's supply chain. Outside sources include economic fluctuation, labor availability, and cost changes of important resources. Preemptive overhaul of key equipments and labor disputes are examples of uncertainty sources originating from inside the warehouse; for example, the Bullwhip effect could be a serious problem in warehouses. The bullwhip effect entails that small fluctuations in customer demand can result in amplified demand and inventory level fluctuations upstream in the supply chain. Mangan et al. [18, p. 129] state " If bullwhip can be controlled, warehousing operations can be run more effectively and efficiently". Wal-Mart takes countermeasure against the bullwhip effect: Individual Wal-Mart stores transmit point-of-sale data from the cash register to corporate information center several times a day and also share information with manufacturers like Procter and Gamble [19]. These improve the visibility of customer demand and inventory movement throughout the supply chain, and are used to schedule shipments among the Wal-Mart distribution centers, stores and suppliers' warehouses. This countermeasure thereby improves operations of Wal-Mart distribution centers.

2.1.3 *Operational uncertainty sources*

Uncertainties from human factors have a short-term impact on order-picking daily planning, consisting of order batching, routing, and picker task assignment. Among these are manual handling risks, labor absence, or specific injuries like musculoskeletal disorders, as reported by Wright and Haslam [20]. Order information distortion caused by order cancelation can affect daily picking planning. Facility daily planning faces uncertainties from equipment failure and facility maintenance. Modern warehouses depend heavily on the proper function of information systems. In this respect, they are sensitive to information infrastructure breakdown, data errors, and errors in the communication with external systems.

We further shed light on the relation between uncertainty source types and impact levels. Based on the examples distributed along the horizontal axis in Fig. 1, we find unpredictable uncertainty sources usually have more strategic or tactical impacts than predictable sources. On the other hand, from examples distributed along the vertical axis in Fig. 1, we find outside sources usually have more strategic or tactical impacts, and inside sources usually have more operational impacts. We therefore conclude that outside, more unpredictable and high variance uncertainty sources usually have more strategic or tactical impacts, and inside, more predictable and low variance uncertainty sources usually have more tactical or operational impacts in warehouse operations.

2.2 Uncertainties of warehouse operations

A typical stochastic system can be divided into arrival, service and departure processes. Classifying warehouse processes by these same three groups helps us to identify appropriate stochastic models as clear distinction among arrival, service, and departure processes exists. We present typical warehouse operations in Fig. 2, which framework is helpful to capture the heterogenous stochastic essence and heterogenous uncertainty sources in different processes.

In the framework of Fig. 2, we view warehouse operations associated with inbound flows as arrival processes, which include product arrivals typically followed by an "inspection and receiving" operation, and order arrivals typically followed by an "acceptance and reject" operation. We view warehouse operations, which create or add value and are main processes to support core warehousing functions and mainly deal with internal flows, as core service processes, including putaway, storage in a reserve area, replenishing the forward area, order-picking, packing, sorting, and accumulating. We group warehouse operations associated with outbound flows as departure processes, which mainly includes inspection and shipping. We further describe uncertainty factors in these three processes and summarize them in Table 1.

Table 1 Warehouse operations with uncertainty factors

Stochastic process	Operation process	Practice	Issues associated with uncertainties
Arrival process	Product arrival	Transportation	Transportation disruption directly affects the arrival process and increases the uncertainty
		Cross docking	Reduce the variability of throughput time by simplifying operation processes
		Receiving scheduling	Reduce uncertainty and improve arrival rate by scheduling the receiving resource like personnel, equipment, dock doors, staging space
		Prereceiving	Reduce uncertainties by capturing information like location assignment and product identification ahead of time
		Receipt preparation	Decrease arrival uncertainties and improve arrival rates by adequate planning
	Order arrival	Customer demand	Seasonality may affect the customer demand, order cancelation will disturb the arrival rate
		Communication	Information system errors between customer and warehouse will increase the uncertainty
Core service process	Putaway	Direct putaway	Direct putaway eliminates staging and inspecting activities
		Directed putaway	Streamline putaway process by maximizing location and cube utilization, and reduce variability of productivity
		Batch and sequenced putaway	An efficient way to stabilize service rate of putaway and reduce the variability of productivity
	Storage	Reserve area storage	Achieve better space utilization and reduce the uncertainty of replenishment shortage
		Forward area Storage	Improve the service rate and reduce the fluctuation of order- picking productivity
	Order picking	Picker-to-parts	Suitable batch and routing polices will improve the service rate. Pick inaccuracy and pick errors increase uncertainty
		Parts-to-picker	An automated conveying system will reduce the uncertainty and improve service rates. Balancing work stations is helpful to streamline processes and reduce uncertainties
	Packaging, accumulation, sortation	Packaging	Packaging order inaccuracy increases the uncertainty and reduces the departure rate
		Accumulation and sortation	Sorter mechanical errors can lead to order inaccuracy in accumulation and sortation and increase uncertainties
Departure process	Shipping	Container loading	Optimization can maximize the cube and utilization of each container and also reduce the uncertainty of utilization
		Staging activity	An automated operation and direct loading can eliminate staging and its uncertainty, and improve the departure rate
		Shipping inaccuracy	Tracking and tracing techniques can decrease the uncertainty of delivery

2.2.1 Uncertainties in arrival processes

There are two main arrival processes in a warehouse system. One arrival process is the physical product arrival. The inventory level at suppliers and transportation will influence the arrival rate and uncertainties of quantities and time in the arrival processes. Wilson [21], for example, investigates the effect of a transportation disruption. Return product arrivals will also increase the arrival variance. After the products have arrived at a warehouse, inspection and receiving operations can lead to congestion or additional delay, and increase the variance of the internal transportation time to the next warehouse operation. Receiving scheduling, prereceiving, and receipt preparation have been applied to decrease the uncertainty of the arrival process. Another arrival process is the order arrival process, determined by customer demand, usually a stochastic variable (e.g., seasonality and sales will affect the customer demand; order cancelation will disturb the arrival rate).

2.2.2 Uncertainties in service processes

Main warehouse operation processes include putaway, storage, order picking, packaging, accumulation and sorting.

1. *Putaway*. Putaway is a critical operation since it determines the efficiency and cost of retrieval, and accounts for about 15% of warehouse operational cost (see Bartholdi and Hackman [22]). Direct putaway eliminates staging and inspection activities. However, without the inspection process, the uncertainties will possibly increase since potential errors cannot be identified in time. Using a Warehouse Management System (WMS), directed putaway can improve efficiency by maximizing location and cube utilization and retrieval productivity. Batched and sequenced putaway can also improve the efficiency.
2. *Storage*. Typical storage consists of forward and reserve storage (not all warehouses have their storage system split in forward and reserve). A forward-reserve area storage strategy will improve the efficiencies of order retrieval and picking. In the reserve area, products are stored in pallet racks or block-stacks to achieve a high space utilization and reduce the uncertainty of replenishment to the forward area. In the forward area with compact size, bin shelving and gravity flow racks are applied to facilitate order picking, and reduce the fluctuation of order-picking productivity. The forward-reserve system is a two-echelon inventory system, and imbalance of the inventory level between reserve and forward areas can lead to a greater variance of throughput (e.g., inventory shortage in the forward area will reduce the order-picking throughput).
3. *Order picking*. Order picking can be divided into two types of systems: picker-to-parts and parts-to-picker. Parts-to-picker systems include automated storage and retrieval systems, using mostly aisle-bound cranes that retrieve one or more unit loads and bring them to a pick position. Such an automated system may streamline the service process, reduce response time, and thereby improve service. In end-of aisle order-picking systems, tailored balancing of humans and machines helps to reduce the throughput variance. In picker-to-parts systems, an order picker walks or drives along the aisles to pick items. Two types can be distinguished: low-level picking and high-level picking. In low-level order-picking systems, the order picker picks requested items from storage racks or bins, while traveling along the storage aisles. Pick inaccuracies, i.e., picking a wrong item or wrong quantity, can increase the uncertainty of the pick service process. High-level (also called man-aboard) order picking is used in warehouses with high storage racks. Order pickers travel to pick locations on board of a lifting order-pick truck or crane, which stops in front of the appropriate pick location and waits for the order picker to perform the pick. If multiple order pickers are used, congestion may occur.
4. *Accumulation, sortation, and packaging*. Accumulation and sortation of picked orders into individual customer orders is a necessary activity if the orders have been picked in batches. Accumulation and sortation processes usually apply mechanical equipments like conveyors and sorters, and man-machine balance will affect the throughput. Mechanical errors like faulty sortations can also cause inaccuracies in accumulation and sortation. Such inaccuracies will increase the uncertainty of the departure process and may reduce the departure rate. During packing, laborers can check whether customer orders are complete and accurate, which can again decrease these uncertainties (see Bartholdi and Hackman [22]).

2.2.3 Uncertainties in departure processes

One of the main uncertainties during shipping stems from shipping inaccuracy, i.e., shipping wrong products to wrong customers, at a wrong time. Errors in electronic messages can further cause or magnify these uncertainties. Other uncertainties in the departure process arise from departure operations like container loading (e.g., wrong order batch, wrong space calculation for containers) and shipment staging (e.g., human factors cause fluctuations in departure rate). Failure of shipping equipment, like trucks, pallet jacks, and counterbalance lift trucks, can also cause uncertainties in this process.

3 Stochastic methods in warehouse operations research

Classical deterministic models assume perfect information is available about the objective function and this information can be used to determine the search direction. However, due to existing uncertainties in warehouse processes (see Table 1; Fig. 1), such perfect information is usually unavailable. Stochastic models provide a means of coping with inherent system noise and coping with models or systems that are dynamic, stochastic, even unstable, or otherwise inappropriate for classical deterministic methods.

Various stochastic models have been applied by warehousing researchers (see Table 2). First, much order-picking work adopts classical probability models, defined by a sample space, events within the sample space, and probabilities of each event, including basic probability models like the binominal, the Bernoulli, the geometric, the hypergeometric models and their derivatives like the urn model. For example, Chew and Tang [11] analyze order-

Table 2 Stochastic models in warehouse operations

Type	Method	Research examples	Problem statement
Classical probability models	Urn models	Chew and Tang [11]	Analyzing the picking systems by urn models
		Le-Duc and De Koster [23]	Travel distance estimation in a 2-block class-based storage strategy warehouse
Classical stochastic models	Renewal process models	Bozer and White [9]	The basic configuration is modeled as a renewal process in end-of-aisle order-picking system
	Markov chain model	Gue et al. [24]	Model the circular picking area with two workers as a Markov process
		Parikh and Meller [25]	Estimating picker blocking in wide-aisle order-picking systems
		Parikh and Meller [26]	Study worker blocking in narrow-aisle order-picking systems when pick time is non-deterministic
Queueing models	Single queueing models	Lee [27]	Analyzing a unit-load AS/RS by a single-server queueing model with two queues and two different service modes
	Queueing networks models	De Koster [10]	Performance approximation of pick-and-pass order-picking systems
		Heragu and Srinivasan [28]	Analysis of manufacturing systems via semi-open queuing networks
	Polling models	Gong and De Koster [1]	A polling-based warehouse dynamic picking system for online retailers
		Bozer and Park [29]	Single-device polling-based material handling systems
Others	Fluid models	Bartholdi et al. [12]	Bucket bridges problem when work is stochastic
	Petri-net models	Hsieh et al. [30]	Present a Petri-net-based structure to describe and model AS/RS operations
		Lin and Wang [31]	Modeling an automated storage and retrieval system using Petri nets

picking operations in a 1-block warehouse, and Le-Duc and De Koster [23] analyze warehousing operations in a 2-block class-based storage strategy warehouse by basic probability models (specifically, the binominal model and urn models) to determine the locations from which articles must be picked in a pick tour and thereby the tour length.

Second, classical stochastic models (like renewal process models, Markov models, martingale models) are also helpful to describe warehousing operation processes; for example, Bozer and White [9] model order-picking operations in an end-of-aisle order-picking system as a renewal process, where an event occurs when both pickers and the storage/retrieval(S/R) machine begin service. Gue et al. [24] model a circular picking area with two workers as a Markov process when they research the effects of pick density in order-picking areas with narrow aisles. Parikh and Meller [25] estimate picker blocking in wide-aisle order-picking systems. Parikh and Meller [26] study worker blocking in narrow-aisle order-picking systems when pick time is non-deterministic.

Third, various queueing models (including single-server queueing models like $M/M/1$ and $M/G/1$, queueing network models, and their derivatives like the polling model) are frequently used in warehousing research. Lee [27] has examined a unit-load AS/RS by a single-server (a S/R machine) queueing model with two queues and two different service modes (storage requests and retrieval requests). According to Bozer and Cho [32], this is the first study using stochastic analysis of a unit-load AS/RS by an analytical method. Queueing networks are also helpful for warehouse modeling. De Koster [10] has researched performance approximation of zoned order-picking systems by a Jackson queueing network. Heragu and Srinivasan [28] have studied manufacturing systems via semi-open queuing networks, which includes the time a customer waits outside the system. Polling models, a special queueing network type, have also drawn the attention of warehousing researchers. Bozer and Park [29] have studied single-device, polling-based material handling systems. Gong and De Koster [1] apply stochastic polling models to a warehouse dynamic order-picking system for an online retailer.

Besides the aforementioned three main types of methods, several other techniques have been introduced to warehouse research; for example, Bartholdi et al. [12] have researched bucket brigades where the work is stochastic by fluid models. Hsieh et al. [30] and Lin and Wang [31] model an automated storage and retrieval system using stochastic Petri nets, and their models can be used to evaluate the performance and optimize control policies. These pioneering researches provide new exploration in warehouse research by stochastic methods.

Table 3 Stochastic analysis in warehouse systems

Type	Method	Research examples	Problem statement
Optimization	Stochastic constrained optimization	Azadivar [33]	To determine the maximum number of storage and retrieval requests in automated warehousing systems
	Perturbation analysis	Gong and de Koster [34]	Approximate optimal order batch sizes in a parallel-aisle warehouse
	Kuhn-Tucker condition	Jucker et al. [35]	The simultaneous determination of plant and leased warehouse capacities for a firm facing uncertain demand in several regions
	Petri-net-based technique	Archetti et al. [36]	Adopted Petri-net models and a stochastic optimization method to study optimal control policies of an AS/RS
Heuristic	Analytical approximation	Bozer and White [37]	Present two efficient heuristic algorithms for design and performance analysis for end-of-aisle order-picking system
Simulation	A tool based on Promodel	Macro and Salmi [38]	Invented a simulation tool to determine warehouse efficiencies and storage allocations based on Promodel
	MC simulation	Rosenblatt and Roll [39]	Analyzing warehouse capacity in a stochastic environment by MC simulation
	Simulation-based regression analysis	Ekren and Heragu [40]	Study the rack configuration of Autonomous Vehicle Storage/Retrieval systems
	Petri-net-based simulation	Hsieh et al. [30]	Propose a Petri-net-based four-layer simulation structure for the AS/RS
	Enumeration	Stadtler [41]	Optimize dimensions for automated warehouse systems by a procedure consisting of enumeration simulation
Others	Determine limiting behavior	Litvak [42]	Determine a limiting behavior of the shorted rotation time needed to collect large orders in a carousel system
	Matrix geometric analysis	Bastani [43]	Analyze closed-loop conveyor systems by M/M/s system and an matrix geometric solution

To analyze these stochastic models, researchers have adopted various methods including optimization, heuristics, and simulation. Some typical examples of each of these methods are listed in Table 3. Stochastic optimization refers to the minimization (or maximization) of a function in the presence of randomness in the optimization process, which applies to one or both of the following conditions. (1) There is random noise in the measurement of the objective function and (2) a random (Monte Carlo) choice is made in the search direction as the algorithm iterates toward a solution. By a stochastic constrained optimization algorithm (a simulation optimization algorithm), Azadivar [33] has determined the maximum number of storage and retrieval requests that can be handled by automated warehousing systems under physical and operational constraints. Jucker et al. [35] develop an efficient algorithm based on Kuhn-Tucker conditions for simultaneously determining the plant and leased warehouse capacities for a firm facing uncertain demand in several geographical regions. Archetti et al. [36] have adopted Petri-net models and a stochastic optimization method to study optimal control policies of an AS/RS.

Heuristic methods have also achieved successful application, for example, Bozer and White [37] present two efficient heuristic algorithms for design and performance analysis of end-of-aisle order-picking operations based on a miniload AS/RS. The algorithm is based on an approximate analytical model developed to estimate the expected picker utilization for a general system configuration.

Simulation has been widely adopted by warehousing researchers. Macro and Salmi [38] analyze the storage capacity and rack efficiency of a medium volume, low stock-keeping unit (SKU) warehouse and a medium volume, large SKU warehouse by Promodel. The model can be applied to simulate various warehouse configurations like bulk floor storage, push-back, flow-through, drive-in, and drive-through racks (for a review of such rack types, see Tompkins et al. [44]). Rosenblatt and Roll [39] have analyzed warehouse capacity in a stochastic environment by Monte Carlo simulation. Ekren and Heragu [40] present a simulation-based regression analysis for the rack configuration of an autonomous vehicle storage and retrieval system (AVS/RS) and give mathematical functions for the rack configuration of an AVS/RS that reflects the relationship between the outputs (responses) and the input variables (factors) of the system. Stadtler [41] optimizes dimensions for automated warehouse systems by a procedure consisting of enumeration simulation. Hsieh et al. [30] propose a Petri-net-based four-layer simulation structure as a general tool to model the operations and evaluate the performance and develop control policies of an AS/RS.

Besides these approaches, some other stochastic analytic methods exist, for example, Litvak [42] determines the limiting behavior of the shortest rotation time needed to collect large orders in a carousel system. Bastani [43] analyzes closed-loop conveyor systems with breakdown and repair of unloading stations by an M/M/s queueing system and provides an approximation of the steady-state probabilities of the system in different operating states by the matrix geometric technique.

Over the last 20 years, we have witnessed a rapid development in stochastic optimization techniques, including stochastic programming and stochastic approximation. From our review, however, we find while simulation and heuristics are widely applied in warehouse research, and stochastic optimization is hardly used.

4 Stochastic applications in warehouse operations

This section examines the application of stochastic methods in main warehouse operations, including storage, order picking, packing, sorting, accumulation, and distribution. For order picking, where many stochastic researches exist, we examine applications in three systems: picker-to-parts systems, parts-to-picker systems, and automated picking systems (see Van den Berg [45]).

4.1 Storage

Storage is a main function of a warehouse, and a large number of papers research it by deterministic methods. Stochastic research in this area, however, is not abundant (compared with order picking). Noteworthy examples include Van den Berg et al. [5], who have studied forward-reserve allocation in a warehouse with unit-load replenishments. Roll et al. [46] present analytical and simulation methods to determine the size of storage containers in a warehouse with an objective to minimize the storage cost. Chang and Wen [47] research the impact on the rack configuration on the speed profile of storage and retrieval machines and present an analytical procedure to obtain the optimal rack configuration in an AS/RS.

4.2 Order picking: picker-to-parts systems

Stochastic research in this area is abundant, for example, Gue et al. [24] build a stochastic throughput model to explore the effect of pick density on order-picking areas with narrow aisles. Roodbergen and Vis [48] apply probability models to the layout design in a picker-to-parts warehouse, with an objective of minimizing picking travel time.

4.3 Order picking: parts-to-picker systems

Parts-to-picker systems in general have a high automation level, and it is convenient to model such systems by stochastic models. For instance, Bozer and Cho [32] derive closed-form analytical expressions for throughput performance of an AS/RS under stochastic demand and also derive an analytical estimate for the expected S/R machine utilization. Park et al. [49] model an end-of-aisle order-picking system as a two-stage cyclic queueing system consisting of one general and one exponential server queue with limited capacity and present closed-form expressions for system performance measures like throughput.

4.4 Order picking: automated picking systems

The number of implementations of automated picking systems is growing. However, only few papers in this area exist. An example is Yu [50], who studies dynamic picking systems. These are systems where not all items have a storage slot in the forward area. Items are moved to the forward area automatically and dynamically when needed from the reserve storage. Since automated picking is a rapidly growing area of interest and since order profiles and storage location selection are stochastic, stochastic modeling of these systems could be explored further.

4.5 Packing, sorting, and accumulation

Although several papers research packing, sorting, and accumulation combined with order picking, few papers focus on these processes using stochastic methods. An exception is Johnson [51], who studies the impact of sorting strategies on automated sortation system performance by a stochastic analytical model. Van Nieuwenhuyse and de Koster [52] consider two sorting policies (pick-and-sort versus sort-while-pick) when they try to evaluate order throughput time in 2-block warehouses with time window batching. Gallien and Weber [53] compare wave-based and waveless picking policies for warehouses with an automated sorter, and provide operational guidelines for order release with an objective to maximize throughput.

4.6 Distribution

Distribution, including internal transport, inbound and outbound shipping, is critical to improve the overall performance of a warehouse system. Research in internal transport is quite abundant. Many papers research vehicle-

Table 4 Stochastic application in warehouse operations

Warehouse operations	Research examples	Problem statement
Storage	Van den Berg et al. [5]	Study forward-reserve allocation in a warehouse with unit-load replenishments
	Roll et al. [46]	Present an approach to determine the size of a warehouse container
	Chang and Wen [47]	Present an analytical procedure to obtain the optimal rack configuration in an AS/RS
Order picking: Picker-to-parts	Gue et al. [24]	Build a stochastic throughput model to explore the effects of pick density on order-picking areas with narrow aisles
	Roodbergen and Vis [48]	Apply probability models to the layout design in a picker-to-parts warehouse
Order picking: Parts-to-picker	Bozer and Cho [32]	Present an analytical result of throughput performance of AS/RS under stochastic demand
	Park et al. [49]	Present queue models for end-of-aisle order-picking systems with buffer positions
Order picking: Automated picking	Azadivar [33]	Maximize the throughput in a computerized automated warehousing system
	De Koster et al. [54]	Consider a newly designed compact three-dimensional AS/RS with automated picking
Packing, sorting, accumulation	Johnson [51]	Study the impact of sorting strategies on automated sortation system performance
	De Jong and Anderson [55]	Study the setting of shelf heights and the distribution of box sizes in two-dimensional shelf packing
	Van Nieuwenhuyse and de Koster [52]	Evaluating order throughput time in 2-block warehouses with time window batching and considering two sorting policies
	Gallien and Weber [53]	Evaluating different order release policies for warehouses with an automated sorter
Distribution	Le-Anh [56]	Study intelligent control of vehicle-based internal transport systems

based internal transport systems like forklifts and AGVs. However, receiving and shipping processes are not especially studied in current literature.

This section is summarized in Table 4. By comparing existing warehouse operations with uncertainties (see Table 1; Fig. 2) and warehouse operations with stochastic studies (Table 4), we can identify interesting academic blanks. These potential research directions will be further presented in Sect. 6.1

5 Current research limitations: model, parameter, process details

In this section, we present limitations of past studies on warehousing, from a stochastic modeling angle. We focus on model inaccuracies referring to limitations from adopting inaccurate mathematical (specially probabilistic or stochastic) models, parameter estimation inaccuracies, and process inaccuracies due to oversimplifying warehouse processes or overlooking important processes.

5.1 Arrival process

The order arrival process is often modeled as a Poisson process (e.g., Lee [27], Axsäter [57, 27]), possibly with a time-varying arrival rate. When the number of orders is relatively large and orders are independent, a Poisson process could be a good approximation. For instance, for warehouses of online retailers, the order size is relatively small and the total number of orders is relatively large, and the Poisson process can be used to approximate the order arrival process (see, e.g. Gong and De Koster [1]). However, the typical Poisson model cannot always accurately describe arrival processes in some application settings.

1. *Model inaccuracies*. The arrival process might not be well modeled by a Poisson process. One reason is, customer orders can be dependent; for example, students in one business school can all order the same book at Amazon. Order arrivals from these students are then correlated. In addition, one order can include several line items, and these line items are dependent. Hence, it is inaccurate to model the order line arrival stream as a Poisson process. Some researchers explicitly model correlated products. Frazelle and Sharp [58] conduct a simulation of a miniload AS/RS where correlated products are stored in the same bins, and report a reduction of 30–40% in the number of retrieval trips compared with that in a setting of random product assignment. A non-homogeneous Poisson process, with a time-dependent rate parameter, may be more suitable for some warehouses. An example of a non-homogeneous Poisson process would be the order arrival rate to the warehouse of an online food retailer, where the arrival rate increases before dinner time and decreases during the remaining parts of the day.
2. *Parameter inaccuracies*. Usually we do not know arrival rates or product correlation coefficients and must estimate them. For a time-varying arrival rate, we

even need to estimate the arrival rate function. In a warehouse, there are a variety of information sources to use for the estimation. However, existing research usually has not provided a convincing justification for the parameter estimation. A more accurate estimation of the order arrival rate from demand data and the product arrival rate from supplier information is needed. One can assume a parametric form for the arrival-rate function, such as linear or quadratic. Massey et al. [59] have explored a method to estimate the coefficients of linear arrival rate functions from nonhomogeneous Poisson process data.
3. *Process inaccuracies*. Both order and product arrival processes may be inaccurately described; for example, in the case of online retailers, customers can legally cancel orders, forming a negative arrival process. For product arrivals, existing research hardly considers arrival uncertainty due to product quality variance, transportation disturbance, and associated rework and product reject flows.

5.2 Service process

Warehouse service processes include picking at a storage position, travel between positions, packing, and other processes. However, unsuitable model selection, impractical parameter estimation, process oversimplification can induce inaccuracies.

1. *Model inaccuracies*. Many order-picking papers assume picking time is constant, but it is not always acceptable to overlook the variance of picking time. Others model service time as a sequence of independent and identically distributed random variables, each with an exponential distribution. Bozer and Cho [32] point out that the coefficient of variation for single command and dual command cycles are known to be considerably less than one in an AS/RS, and “exponentially distributed S/R service times produce results inconsistent with simulation ”. We can also find other cases where the actual service-time distribution is not exponential; for example, pick time can depend on item types (e.g., the pick time for large items can be longer than for small items) and ergonomic factors (e.g. the pick time of laborers can become longer due to fatigue, see Larco et al. [60]).
2. *Parameter inaccuracies*. Parameters (e.g., pick rate) may be inaccurately estimated. Examples causing parameter inaccuracies include ergonomic factors, which may cause productivity to decrease over time, dependent on, for example, the frequency and length of short breaks, and item heterogeneity which causes the variance of service time. Better parameter estimation can be obtained by analyzing historical data or ergonomic experiments.
3. *Process inaccuracies*. Existing literature often pays no attention to several important factors in service processes. First, in several queueing models studying order picking, capacity limitations, including order picker capacity and cart capacity, are overlooked. But this capacity limitation changes the pick process. The second noticeable problem is order correlation, which will affect the order pickers’ behavior and picking process, and make an exponential or constant serve time assumption unrealistic. Finally, most literature overlooks the congestion problem, which has a significant effect on service processes; for example, in the forward-reserve problem (see Van den Berg et al. [5]), concurrent replenishments may cause congestion in the order-picking process, and replenishments that have not been carried out timely lead to delays in the pick completion time. Gue et al. [24] are among the first to consider the factor of congestion in order-picking systems, and describe this process and its impacts more accurately.

5.3 Departure process

The departure process is often modeled as a Poisson process or even overlooked. However, the departure process is directly associated with customer satisfaction. It is important to enhance warehouse performance by improving the departure process.

1. *Model inaccuracies*. Departure streams are possibly dependent; for example, departures to the same destination are highly correlated since one customer may request multiple orders, or even multiple shipments. Furthermore, irregularity uncertainties exist during the whole process of departure; for example, irregular traffic disruption and congestion will disturb shipments and influence the supply chain, including warehouse operations (see, e.g., Sankaran et al. [61] and Wilson [21]). Shipping inaccuracies (e.g., wrong product, wrong destination), which frequently occur in practice, will disturb the departure process (e.g., by changing the destination during the shipping process). In that case, a Poisson process may be unsuitable to model the departure process.
2. *Parameter inaccuracies*. Parameter inaccuracies exist also in warehouse shipping. Departure parameter estimation will benefit by explicitly considering transportation distortion and shipping inaccuracy. It can be done by analyzing historical data.
3. *Process inaccuracies*. Existing research often assumes the departure process to be a Poisson process, which

may not accurately capture its essence. Batch delivery, a typical departure process in practice, cannot be described by a classical Poisson process. Furthermore, customers may be not satisfied with a shipped product and return it, a process typical for online retailers. Therefore, a return flow may exist in the departure process.

6 Future direction

In this section, by comparing and summarizing the review of Sects. 2, 3, and 4, we present promising research directions with a potential to be applied to warehouse operations. We focus on recent warehousing phenomena that have received little academic attention, like warehouses with an online front desk, self-storage warehouses, and third-party warehouses, and stochastic research directions which can grasp the inherent decision essence and variability structure in warehouse operations.

6.1 Application issues

By comparing Table 1, which presents existing warehouse operations with uncertainties, and Table 4, which presents warehouse operations with stochastic studies, we can identify warehouse operations with uncertainties but not yet modeled by stochastic methods.

6.1.1 Warehouse receiving management

We could not find papers explicitly employing stochastic models for receiving processes (see Table 4). However, receiving is an important issue for warehouse operations (see Table 1) and several interesting research opportunities exist here. The first opportunity is to study storage decisions for returned products. Many online retailers face this problem. To speed up return processes, it may be helpful to not consolidate them with existing stock, but to store them at separate locations. This will be at the expense of more space needed, which in turn may also increase average storage, retrieval, and travel time. The objective is to make the proper decision to take this trade-off into account. Furthermore, warehouse receiving operations (e.g., decentralized receiving, prereceiving) in uncertain environments call for further research by stochastic methods; for example, Yano et al. [62] conduct a successful research on decentralized receiving operations (receiving occurs not at one or two clusters of receiving docks but at multiple locations) by a mixed integer nonlinear optimization formulation with the objective of minimizing total cost of facilities and labor. Splitting receiving operations over multiple areas can reduce congestion, but usually requires more resources and reduces resource flexibility. It could be interesting to consider this trade-off when product arrival times and order patterns are random.

6.1.2 Warehouse revenue management

Comparing Fig. 2 and Table 4, we find that order acceptance and rejection have been overlooked by past literature. A warehouse manager can reject an order to maximize the revenue. For instance, Shurgard (see shurgard.eu in the EU, and publicstorage.com in the USA), an international corporation providing third-party warehouse services, uses stochastic revenue management in allocating storage space to clients. Customers that cannot be accommodated with a space size of their choice can either be rejected or upgraded to a larger space. It is interesting to explore a new warehouse design method to fit market segments and accommodate volatile demand in order to maximize revenue. Another promising topic is to study how to improve the revenue of self-storage warehouses by optimizing storage scheduling decision, for a self-storage warehouse, facing a set of reservations for homogeneous or heterogeneous storage units over a certain time horizon with revenue rewards. The warehouse operation manager has to decide which storage requests to accept and schedule them in different storage units to maximize the revenue. These warehouse operations can be modeled as scheduling independent multiprocessor tasks with given start and end times, with an objective to maximize total revenue.

6.1.3 Order-picking management

Although order picking has been researched quite extensively, stochastic models provide an opportunity to particularly model the impact of variability on performance; for example, in pick-and-pass order- picking systems, performance is influenced by the buffer size (expressed in number of customer totes) at the picking stations. In order to estimate this effect use can be made of (approximate) analysis of queuing networks with blocking. These methods may also provide insight not only in mean performance (throughput, station utilization, lead time) but also in variability of lead time, which is important if the orders have to meet fixed due times for truck departure.

6.1.4 Warehouse shipping management

Only few papers deal with outbound material flows (e.g. Yu and Egbelu [63]), mainly in a deterministic environment. Shipping operations are often overlooked. Frazelle [2] argues that while it is helpful for increasing logistic efficiencies, design and selection of shipping containers (including cartons, totes, pallets, trailers, ocean containers, rail cars, and air containers) throughout the entire supply chain is “one of most neglected opportunities”. Nevertheless, many important shipping problems exist; for example, how to allocate products to be shipped to different shipping docks. With the increase of innovative warehouse shipping operations like automated pallet loading, automated outbound weight checking, advanced shipping notice preparation, dock assignment optimization (see Frazelle [2]), and the increase of uncertainties from these operations, it could be an interesting topic to explore shipping operations by stochastic methods.

6.1.5 *Real-time response systems*

Real-time response constitutes one of most vibrant warehouse research fields. To shorten response time (from order notification to the shipping to customers, see Fig. 2), new techniques have been introduced, such as online picking (using, for example, pick-by-voice), RFID systems, and fluid shipments (for more information on these techniques, see De Koster [3]). In a dynamic environment, decision-makers have insufficient time to collect information, and therefore, the negative effect of uncertainties is larger. Deterministic models cannot capture the inherent uncertainty in these systems. Stochastic models might be used to model these systems, to measure the performance of real-time order processing in a stochastic environment, and to optimize these systems.

6.2 Methodology issues

By comparing stochastic methods (e.g., see Yao [64]) with currently used stochastic methods in warehouse operations (see Tables 2, 3), and considering current developments in warehousing practice, we can identify promising methodological research directions.

6.2.1 *Stochastic networks applications*

Queueing networks have been applied in warehouse research to some extent (e.g., Gong and De Koster [1], Meng and Heragu [1, 65]). However, more general stochastic networks, one of the main recent exploration directions in stochastic research [64], have appeared to be promising in the operations and manufacturing areas, and can be explored further in warehousing. For instance, stochastic fluid models can be used to represent customers in a service facility, or jobs on the work floor [66]. Stochastic networks are potential tools to handle tough warehouse problems like large order flows in multiple work stations, multi-stage warehouse processes, and dynamic scheduling problems.

6.2.2 *Stochastic programming applications*

From our literature review, one of the most obvious blanks of stochastic methodology in warehousing research is stochastic programming (for an introduction, see Birge and Louveaux [67]). We could not find an application of this important stochastic analytical method in warehouse research (for an introduction to this research stream, see stoprog.net). However, it benefits warehouse optimization problems; for example, many papers (e.g., Van den Berg et al. [5], Karasawa et al. [6]) employ integer programming applications in warehousing since warehouse managers face many integer decision variables like batch sizes and the number of zones to use. But due to risks and uncertainties in these warehousing decisions (see Table 1), stochastic integer models are closer to practice. While deterministic models only consider the first moment of measurements (e.g., the objective) and can cause significant errors, stochastic models can research higher moments of measurements and capture more abundant information. Recently, polynomial time algorithms for stochastic integer programming problems have seen increasing research attention [68]. They might be used for various problems, including product assignment, storage space allocation, the optimal batch size, due time realization, and optimal zone problems.

6.2.3 *Stochastic combinatorial problems*

Stochastic combinatorial optimization is a highly promising method in warehouse research, especially the stochastic traveling salesman and stochastic knapsack problems. The application of stochastic traveling salesman models has constituted a main foundation in the logistics field [69–71] and can be applied to the internal picking routing problem in warehouses. Another promising method is the stochastic knapsack model (see [72, 73]), which can be applied to the warehouse storage space allocation problem. Furthermore, Kleywegt and Papastavrou [74] and Kleywegt and Papastavrou [75] explore dynamic and stochastic knapsack problems. These methods may be applied to allocate warehouse storage space in static and dynamic environments.

6.2.4 *Robust optimization*

An alternative, albeit more pessimistic, approach to model uncertainties in objectives and constraints and model parameters in general, is robust optimization. This approach aims at minimizing the worst-case effects and is particularly appropriate if stochastic distributions are just unknown (see Ben-Tal and Arkadi Nemirovski [76]). Robust optimization can be applied to many warehousing problems, like storage slotting for unknown demand, allocation of trucks to dock doors for unknown arrival times, and order-picking system design for unknown demand.

Stochastic researches could also shed light on other questions like the optimization of dynamic storage and putaway systems in a stochastic environment, the optimal zone problem by using stochastic integer programming, and the optimal batch size problems by infinitesimal perturbation analysis techniques.

7 Concluding remarks

In this paper, we present a literature review on stochastic modeling and analysis of warehouse operations. We identify strategic, tactical, and operational uncertainty sources, and systematically explore uncertainties of arrival, service, and departure processes in a warehouse. These uncertainties explain why researchers might resort to stochastic rather than deterministic models in some uncertain environments.

In the past, deterministic models have achieved successful applications in warehouse research. Researchers may be inclined to think stochastic methods are limited to classical probability models. However, we find not only a substantial number of stochastic applications, but also a great variation in methods. These improve our understanding on warehouse research.

Nevertheless, we find while stochastic models are potentially efficient tools for warehouse research, the application of stochastic methods in warehouse research could be explored further. We identify several directions highly relevant to practice and largely unexplored in warehouse literature, including real-time response models, warehouse revenue management, receiving management, and shipping management which can be explored by methods like stochastic programming, stochastic combinatorial modeling, and stochastic network modeling. Although many problems have been solved in practice in a heuristic fashion, there are still academic blanks, particularly into optimal and robust approaches.

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References

1. Gong Y, De Koster MBM (2008) A polling-based dynamic order picking system for online retailers. IIE Trans 40:1–14
2. Frazelle E (2001) World-class warehousing and material handling. McGraw-Hill, New York
3. De Koster MBM, Le-Duc T, Roodbergen KJ (2007) Design and control of warehouse order picking: a literature review. Eur J Oper Res 182:481–501
4. Ratliff HD, Rosenthal AS (1983) Order-picking in a rectangular warehouse: a solvable case of the traveling salesman problem. Oper Res 31:507–521
5. Van den Berg JP, Sharp GP, Gademann AJRM, Pochet Y (1998) Forward-reserve allocation in a warehouse with unit-load replenishments. Eur J Oper Res 111:98–113
6. Karasawa Y, Nakayama H, Dohi S (1980) Trade-off analysis for optimal design of automated warehouses. Int J Syst Sci 11:567–576
7. Lowe TJ, Francis RL, Reinhardt EW (1979) A greedy network flow algorithm for a warehouse leasing problem. AIIE Trans 1:170–182
8. White JA, Francis RL (1971) Normative models for some warehouse sizing problems. AIIE Trans 3:185–190
9. Bozer YA, White JA (1990) Design and performance models for end-of-aisle order picking systems. Manage Sci 36:852–866
10. De Koster MBM (1994) Performance approximation of pick-to-belt orderpicking systems. Eur J Oper Res 72:582–597
11. Chew E, Tang L (1999) Travel time analysis for general item location assignment in a rectangular warehouse. Eur J Oper Res 112:582–597
12. Bartholdi JJ, Eisenstein DD, Foley RD (2001) Performance of bucket brigades when work is stochastic. Oper Res 49:710–719
13. Gu J, Goetschalckx M, McGinnis L (2007) Research on warehouse operation: a comprehensive review. Eur J Oper Res 177:1–21
14. Van den Berg JP, Zijm WHM (1999) Models for warehouse management:classification and examples. Int J Prod Econ 59:519–528
15. Cormier G, Gunn E (1992) A review of warehouse models. Eur J Oper Res 58:3–13
16. Chopra S, Sodhi M (2004) Managing risk to avoid supply-chain breakdown. Sloan Manage Rev 46:53–61
17. Ghiani G, Laporte G, Musmanno R (2004) Introduction to logistics systems planning and control. Wiley, West Sussex
18. Mangan J, Lalwani C, Butcher T (2008) Global logistics and supply chain management. Wiley, London
19. Grean M, Shaw M (2002) Supply-chain integration through information sharing: Channel partnership between Wal-Mart and Procter & Gamble. Research note of Center for IT and e-Business Management, University of Illinois, Urbana-Champaign
20. Wright EJ, Haslam RA (1999) Manual handling risks and controls in a soft drinks distribution centre. Appl Ergon 30:311–318
21. Wilson MC (2007) The impact of transportation disruptions on supply chain performance. Transp Res Part E Logist Transp Rev 43(295–320):43, 295–320
22. Bartholdi JJ, Hackman ST (2006) Warehouse and distribution science. http://www.warehouse-science.com
23. Le-Duc T, De Koster MBM (2005) Travel distance estimation and storage zone optimization in a 2-block class-based storage strategy warehouse. Int J Prod Res 43:3561–3581
24. Gue KR, Meller RD, Skufca JD (2006) The effects of pick density on order picking areas with narrow aisles. IIE Trans 38:859–868
25. Parikh PJ, Meller RD (2009) Estimating picker blocking in wide-aisle order picking systems. IIE Trans 41:232–246
26. Parikh PJ, Meller RD (2010) A note on worker blocking in narrow-aisle order picking systems when pick time is non-deterministic. IIE Trans 42:392–404
27. Lee F (1997) Performance analysis for automated storage and retrieval systems. IIE Trans 29:15–28
28. Heragua S, Srinivasan M (2011) Analysis of manufacturing systems via single-class, semi-open queuing networks. Int J Prod Res 49(2):295–319

29. Bozer YA, Park JH (1999) Expected move request waiting times in single-device, polling-based material handling systems. IIE Trans 33:55–63
30. Hsieh S, Hwang J-S, Chou H-C (1998) A Petri-net-based structure for AS/RS operation modelling. Int J Prod Res 36:3323–3346
31. Lin S-C, Wang H-P (1995) Modeling an automated storage and retrieval system using Petri nets. Int J Prod Res 33:237–260
32. Bozer YA, Cho M (2005) Throughput performance of automated storage/retrieval systems under stochastic demand. IIE Trans 37:367–378
33. Azadivar F (1986) Maximization of the throughput of a computerized automated warehousing-system under system constraints. Int J Prod Res 24:551–566
34. Gong Y, de Koster MBM (2009) Approximate optimal order batch sizes in a parallel-aisle warehouse. In: Bertazzi L, Grazia Speranza M, Nunen JAEE (eds) Lecture notes in economics and mathematical systems, vol 619, pp 175–194
35. Jucker JV, Carlson RC, Kropp DH (1982) The simultaneous determination of plant and leased warehouse capacities for a firm facing uncertain demand in several regions. IIE Trans 14:99–108
36. Archetti F, Sciomachen A, Gaivoronski A (1991) Optimal control policies for automated storage/retrieval system using PN models and stochastic optimization. In: J Billington et al (eds) Proceedings of the fourth international workshop on Petri nets and performance models, IEEE Computer Society, Melbourne, pp 258–267
37. Bozer YA, White JA (1996) A generalized design and performance analysis model for end-of-aisle order-picking systems. IIE Trans 28:271–280
38. Macro JG, Salmi RE (2002) A simulation tool to determine warehouse efficiencies and storage allocations. In: Yucesan E, Chen C-H, Snowdon JL Charnes JM (eds) Proceedings of the 2002 winter simulation conference, pp 1274–1281
39. Rosenblatt MJ, Roll Y (1988) Warehouse capacity in a stochastic environment. Int J Prod Res 26:1847–1851
40. Ekren B, Heragu S (2010) Simulation based regression analysis for the rack configuration of an autonomous vehicle storage and retrieval system. Int J Prod Res (48):6257–6274
41. Stadtler H (1996) Optimal dimensions for automated storage/retrieval systems. In: Progress in material handling research, the material handling industry of America, Charlotte, NC, pp 551–571
42. Litvak N (2006) Optimal picking of large orders in carousel systems. Oper Res Lett 34:219–227
43. Bastani AS (1990) Closed-loop conveyor systems with breakdown and repair of unloading stations. IIE Trans 22:351–360
44. Tompkins JA, White JA, Bozer YA, Frazelle EH, Tanchoco JMA (2003) Facilities planning. Wiley, NJ
45. Van den Berg JP (1999) A literature survey on planning and control of warehousing systems. IIE Trans 31:751–762
46. Roll Y, Rosenblatt MJ, Kadosh D (1989) Determining the size of a warehouse container. Int J Prod Res 27:1693–1704
47. Chang D-T, Wen U-P (1997) The impact on rack configuration on the speed profile of storage and retrieval machine. IIE Trans 29:525–531
48. Roodbergen K, Vis IFA (2006) A model for warehouse layout. IIE Trans 38:799–811
49. Park BC, Frazelle EH, White JA (1999) Buffer sizing models for end-of-aisle order picking systems. IIE Trans 31:31–38
50. Yu M (2008) Enhancing warehouse performance by efficient order picking. ERIM Ph.D. series research in management, vol 139
51. Johnson ME (1998) The impact of sorting strategies on automated sortation system performance. IIE Trans 30:67–77
52. Van Nieuwenhuyse I, de Koster RBM (2009) Evaluating order throughput time in 2-block warehouses with time window batching. Int J Prod Econ 121:654–664
53. Gallien J, Weber T (2010) To wave or not to wave? Order release policies for warehouses with an automated sorter. Manuf Service Oper Manag 12(4):642–662
54. De Koster MBM, Le-Duc T, Yu Y (2008) Optimal storage rack design for a 3-dimensional compact AS/RS. Int J Prod Res 46(6):1495–1514
55. De Jong MJ, Anderson E (1995) The setting of shelf heights and the distribution of box sizes in two-dimensional shelf packing. Int Trans Oper Res 2:181–186
56. Le-Anh T (2005) Intelligent control of vehicle-based internal transport systems. PhD thesis, RSM Erasmus University, The Netherlands
57. Axsäter S (1995) Approximate evaluation of batch-ordering policies for a one-warehouse, N non-identical retailer system under compound poisson demand. Naval Res Logist 42(5):807–819
58. Frazelle EA, Sharp GP (1989) Correlated assignment strategy can improve order-picking operation. Ind Eng 4:33–37
59. Massey W, Parker G, Whitt W (1996) Estimating the parameters of a nonhomogeneous Poisson process with a linear rate. Telecommun Syst 5:361–388
60. Larco JA, De Koster R, Roodbergen KJ, Dul J (2010) Managing warehouse efficiency and worker discomfort through enhanced storage assignment decisions. ERIM working paper
61. Sankaran J, Gore A, Coldwell B (2005) The impact of road traffic congestion on supply chains: insights from Auckland, New Zealand. Int J Logist Res Appl 8(2):159–180
62. Yano CA, Bozer Y, Kamoun M (1998) Optimizing dock configuration and staffing in decentralized receiving. IIE Trans 30:657–668
63. Yu W, Egbelu PJ (2008) Scheduling of inbound and outbound trucks in cross docking systems with temporary storage. Eur J Oper Res 184:377–396
64. Yao D (1994) Stochastic modeling and analysis of manufacturing systems, Series in Operations Research. Springer, New York
65. Meng G, Heragu SS (2004) Batch size modeling in a multi-item, discrete manufacturing system via an open queuing network. IIE Trans 36:743–753
66. Chen H, Mandelbaum A (1994) Hierarchical modeling of stochastic networks, Part I: fluid models. In: Yao D (ed) Stochastic modeling of manufacturing systems. Springer, New York, pp 47–105
67. Birge JR, Louveaux F (1997) Introduction to stochastic programming. Springer, New York
68. Klein Haneveld W, van der Vlerk MH (1999) Stochastic integer programming: general models and algorithms. Ann Oper Res 85:39–57
69. Bertsimas DJ, Van Ryzin G (1991) A stochastic and dynamic vehicle routing problem in the Euclidean plane. Oper Res 39:601–615
70. Bertsimas DJ (1992) A vehicle routing problem with stochastic demand. Oper Res 40:574–585
71. Bertsimas DJ, Van Ryzin G (1992) Stochastic and dynamic vehicle routing problem in the Euclidean plane with mutiple capacitated vehicles. Oper Res 41:60–76
72. Ross KW, Tsang D (1989) The stochastic knapsack problem. IEEE Trans Commun 34:47–53
73. Ross KW, Yao DD (1990) Monotonicity properties of the stochastic knapsack. IEEE Trans Inf Theory 36:1173–1179
74. Kleywegt AJ, Papastavrou JD (1998) The dynamic and stochastic knapsack problem. Oper Res 46:17–35
75. Kleywegt AJ, Papastavrou JD (2001) The dynamic and stochastic knapsack problem with random sized items. Oper Res 49:26–41
76. Ben-Tal A, Arkadi Nemirovski A (2002) Robust optimization methodology and applications. Math Program Ser B 92:453–480